System Dynamics Modeling and I*I Tech Development Revisited

II-IIS-MISCOULECT#Epilogue 2

21 March 2007

…………In preparation

References…

- These epilogue lectures have extensively referred to following literature:
  - R. G. Coyle, “Management System Dynamics”, A Wiley-Interscience Publication, John Wiley & Sons, NY, 1977,
  - George P. Richardson and Alexander L. Pugh III, “Introduction to System Dynamics Modeling with DYNAMO”, Productivity Press, Cambridge, Massachusetts, Boston, US, 1981,
  - CIIR Research papers,
  - Referrals on CIIR Web Site CIIR   Systems”,

Objective of A System Dynamics Analysis

- In general a System Dynamics Analysis has a two fold objective:
  - Explaining the system's behavior in terms of structure and policies,
  - Suggesting changes to structure, policies, or both, which will lead to an improvement in the behavior,

    or

    Suggesting changes to structure and policy in a small system, which will enable it to survive

- Figures to follow show difference between open-loop and closed-loop systems diagrammatically.

Fig.(1): An Open – Loop System

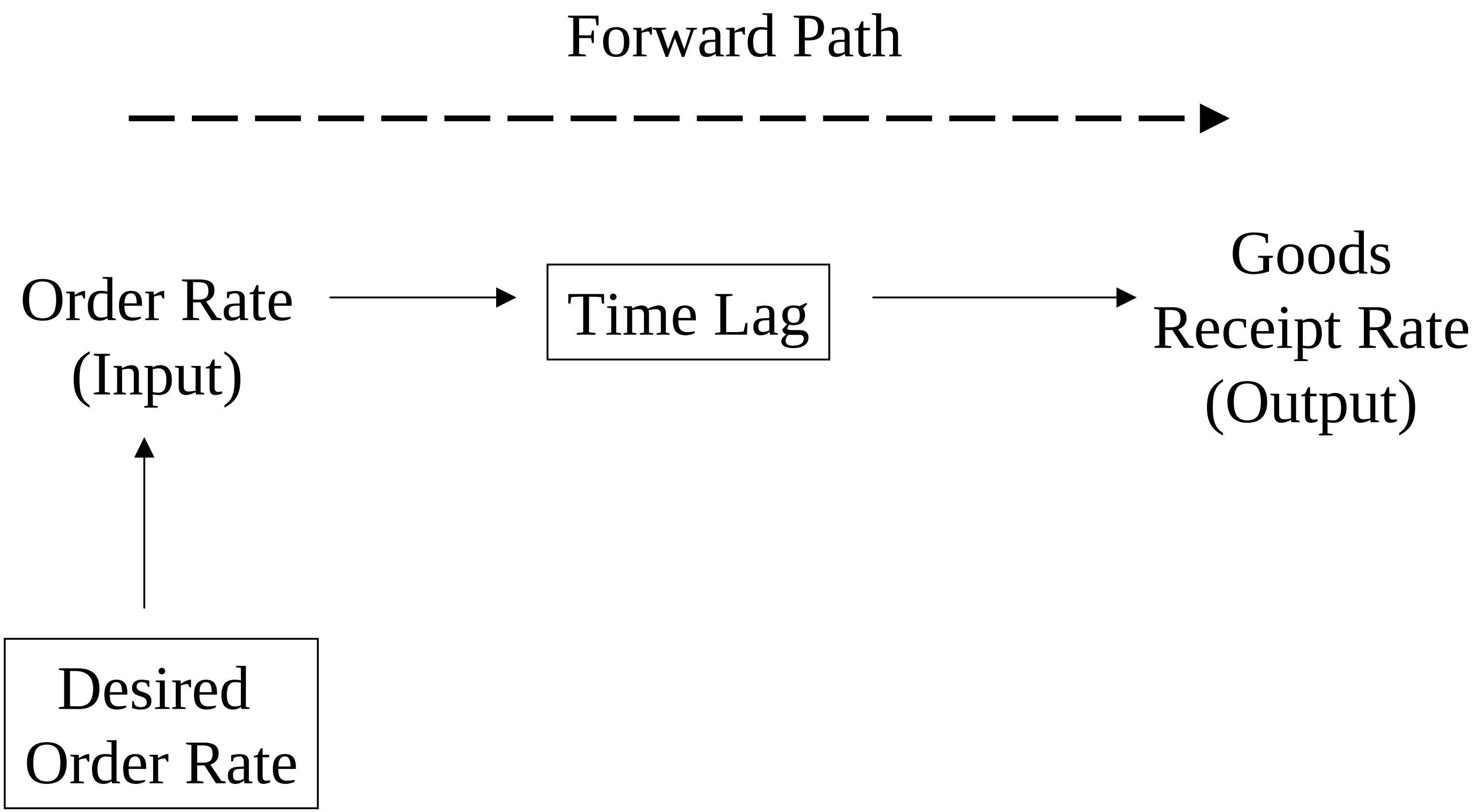
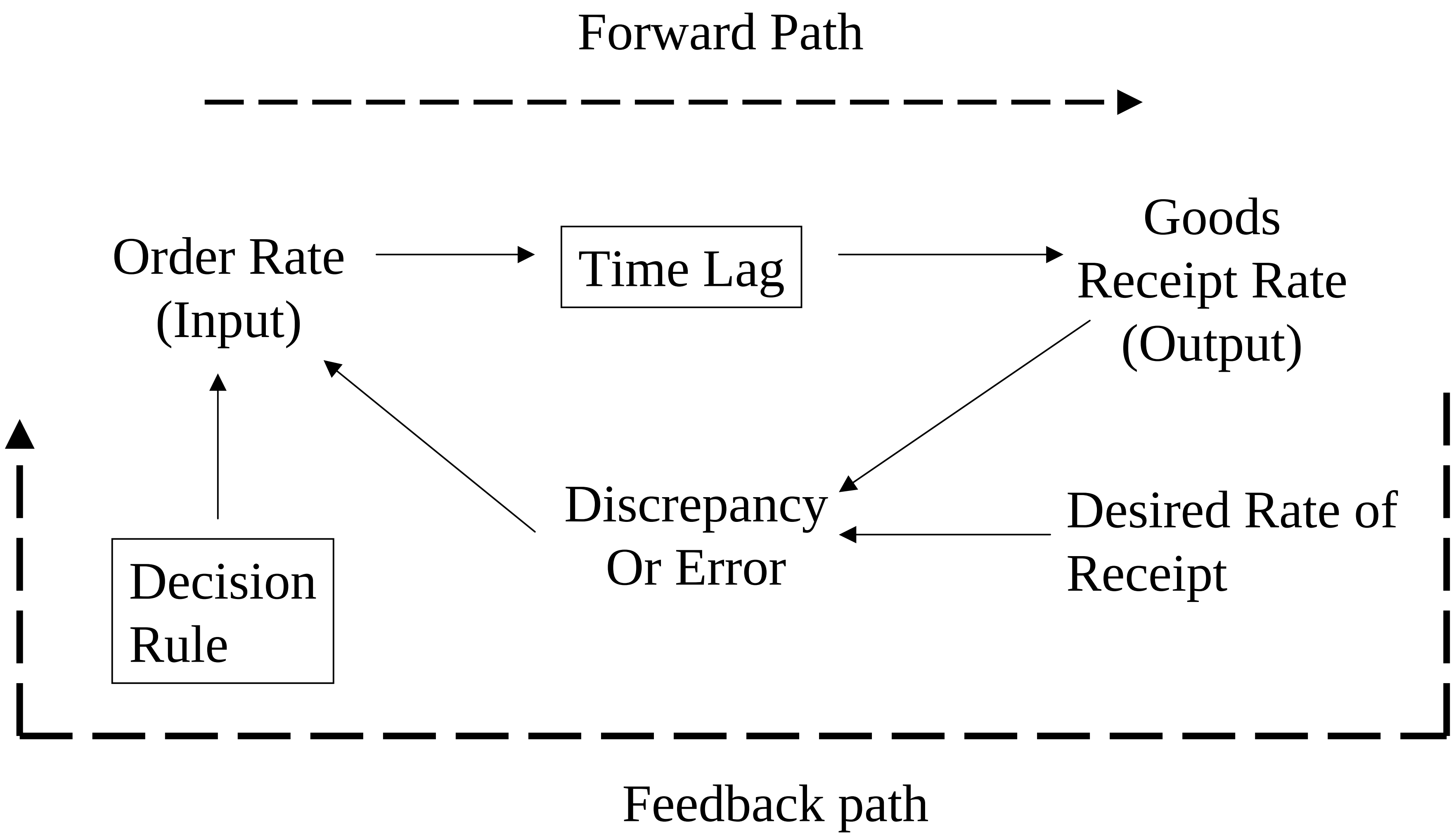


Fig. (2): A Closed – Loop System



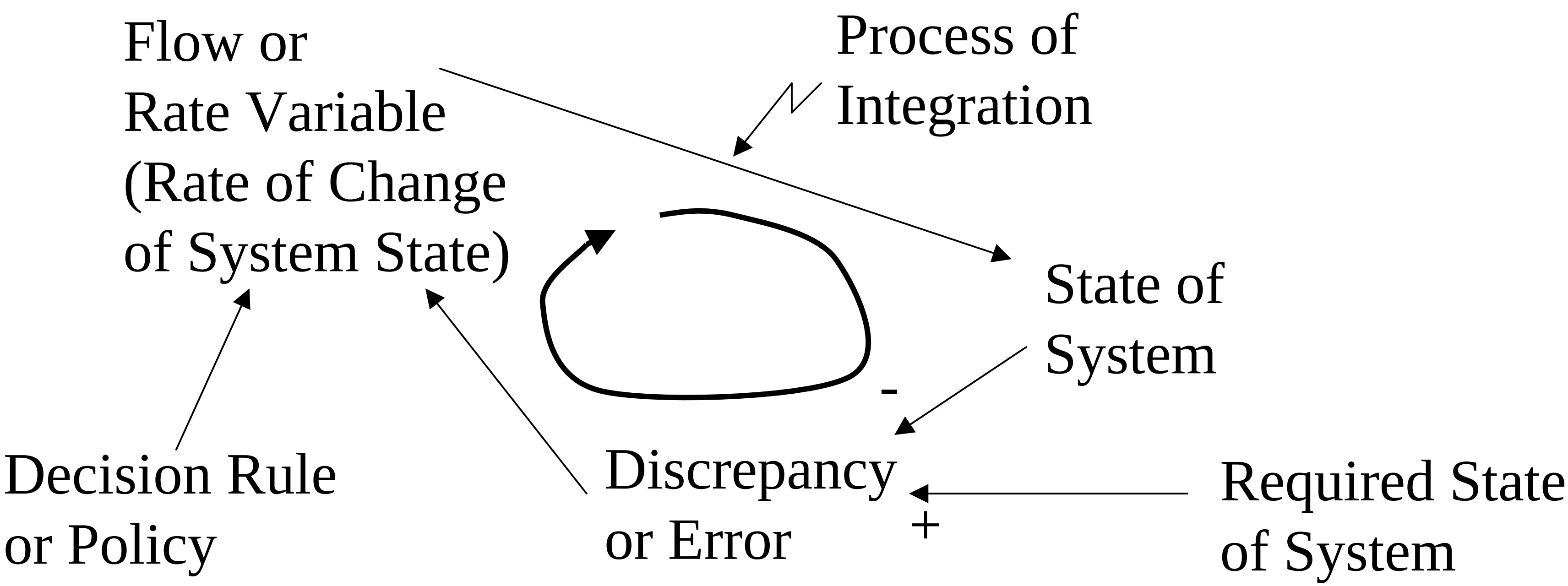
Significance of Feedback Loop

- The distinguishing feature of the closed system is a FEEDBACK PATH of information, choice and action, connecting the output to to the input.
- This creates closed chain of cause-and-effect, or FEEDBACK LOOP, consisting of the physical flow of goods and associated feedback path of information.

- The important thing about feedback loops id that they are the cause of dynamic behavior.
- An open loop system *can* have dynamics arising from its response to external changes.
- The dynamics of a closed – loop system on the other hand, are created by its attempts to control itself in the face of external variations.

- The structure of a closed-loop system includes the way in which its flows of materials and/or information are connected *and* the way in which these flows are codified or transformed by delays and decision rules embedded in the loop.
- Obviously, the key to improving the dynamics of a closed-loop system will lie in arranging its structure, either by altering the connectedness of its flows and/or managing its decision rules.

Fig. (3): The general Structure of Decision Process



→ Flow of information or control

➡ Direction and structure of feedback loop

Microstructure of Feedback Loops

- Need now is to develop a mathematical language for description of the microstructure of the feedback loops.
- Let:
  - INV = (i. e., denote) Amount of inventory,
  - PRODRATE = (denote) Rate of production over a period of time, and
  - CONSRATE = (denote) Rate of consumption over a period of time.

- Further, let:
  - J = Immediate past instant (point) of time,
  - K = Present instant (point) of time,
  - L = Immediate future instant (point) of time, and
  - DT = Time interval between J and K (denoted as “JK”)

    = Time interval K and L (denoted as “KL”)
- Then we have the followings:
  - INV.K = Inventory value at present time “K”,
  - PRODRATE.KL = Production rate during “KL” time interval

  and so on.

- For an inventory-production system, we can now have a Conservation Equation and the same is given here:

  INV.K=INV.J+ DT x (PRODRATE.JK-CONSRATE.JK)

Major Principles about Feedback Loop Microstructure

- The state of the system at any given time is is described by the levels (i.e., stocks or accumulations).
- Levels, and only levels, create rates (i. e., flows) and the rates in turn create new values of the levels, thereby moving the system state with the passage of time.

- There are necessary relationships between levels and rates, which mean that the type of particular variable depends on its position in the feedback loop relative to other variables and not merely on its units of measurement.
  - Thus, the fact that a variable is measured in, say tons/week *does not necessarily mean that it has to be a rate*, a level may very well have such dimensions.
    - For example, if acceleration (measured in ft^2/second) of a vehicle is the rate variable, then by integrating it we get the value of the vehicle velocity, which now is a level variable and the system state even when its dimensions are ft./second.

- Levels and rates are necessary and sufficient to provide microstructure of feedback loops and thus of the system as a whole. In practice, it is convenient to allow for the presence of a third variable type, the Auxiliary, because systems often involve complicated processes by which the decision rules give the rates from the values of the levels, and the intermediate variables may be useful.
  - For example, in Figure (3), variable ‘discrepancy or error’ is an intermediate variable.

- Such intermediate, or auxiliary, variables are not strictly necessary, because they are subsidiary parts of the rate equation, and could be substituted into them. However, this make the rat equations very cumbersome and the auxiliaries are a considerable practical simplification.
- An auxiliary is denoted as say AUX.K to show that it is a subsidiary value, existing only at “K”, and used in determining rats for the period “KL”.

- Three other types of quantities are also found in system dynamics models.
  - *Constants* specifying parameter values, e.g., the number of weeks of of average sales which form target inventory,
  - Supplementary variables indicating, to the analyst, the performance of the system, e.g., Cumulative Lost Sales, but do not form the part of the system itself, and
  - *Initial conditions* for all the levels and some rates.

Relationships in the Structure of Decision Process

- Figure (3) gives the structure of the decision process.
- The above structure and the models of information flow and accumulation mean that a system consists of a collection of feedback loops, each of which include variables of different types.
- There are necessary relationships about the way in which variable types are connected by the loops.

- The relationships leading to formulation of the structures of a system are:
  - A level in a loop can only be preceded by by a rate.
  - Levels are the means by which the system acts to control itself though its decision rules, which produce the flows or rates.
  - The decision rule may be so complex as to call for the use of auxiliary variables, therefore, *a level may be followed by an auxiliary or a rate*.

- An auxiliary is a step in the determination of a rate, but it may not be the only one and, therefore, an auxiliary may be followed by another auxiliary, or by a rate.
- A rate has to be accumulated into a level a rate must be followed by a level.
- A level may not directly affect another level. The connection between two levels can only be through an intervening structure of at least one rate and, possibly, one or more auxiliaries.

Axiom of System Structure

- A feedback loop *must* contain at least one level *and* at least one rate.
- A closed chain of auxiliaries alone would not be a feedback loop and would have no dynamic properties.
- Levels are usually easily recognized as they are the results of accumulation, e.g., stocks, or of averaging. They are the variables whose values would not drop to zero if all the flows in the system were stopped.

- Rates are the flows in the system. They cannot be observed and their values used except by accumulation or averaging – both of which create levels. Rates can only flow into and out of levels except when they feed a *Delay*, which as we shall see, involves hidden intermediate levels.
- Auxiliaries give the fine structure of the system in the way in which the levels govern future rates. They are observable, because they stem from levels, and they can be used to create new system structure if they wish to do so.

DELAYS

- A common element in a system are delayed flows, which are very important in determining the system’s behavior.
- There are material delays and information delays.
- Examples of material delays:
  - Production is ordered at some time, but completed at a later time.
  - Workers are recruited to the labor force but they become fully trained later.
  - Production capacity and commissioning are separated by a time lag and so on.

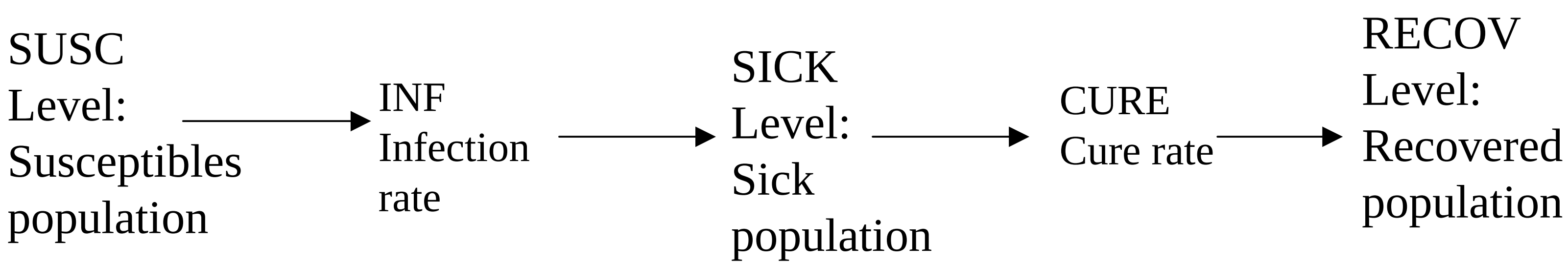
- In susceptible people, the disease incubates,which takes some time, and then after some time delay some of them become sick.

- Examples of information delays:
  - Just as material takes time to flow from one point in a system to another, so does information.
  - The president of a company does not know today's sales rate for one of its branches,
  - An economist does not know the gross national production rate for today,

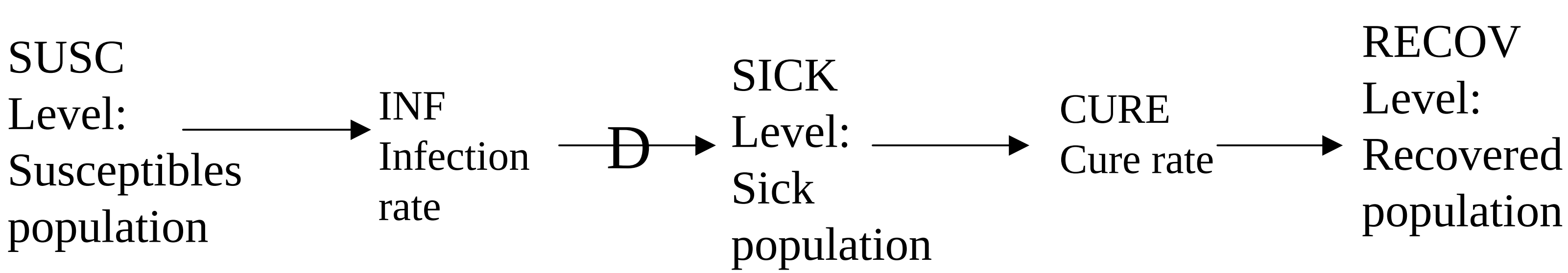
– Delhite’s perception of the job market in Banagalore is bound to lag behind the true picture.

Microstructure of (Material) Delay

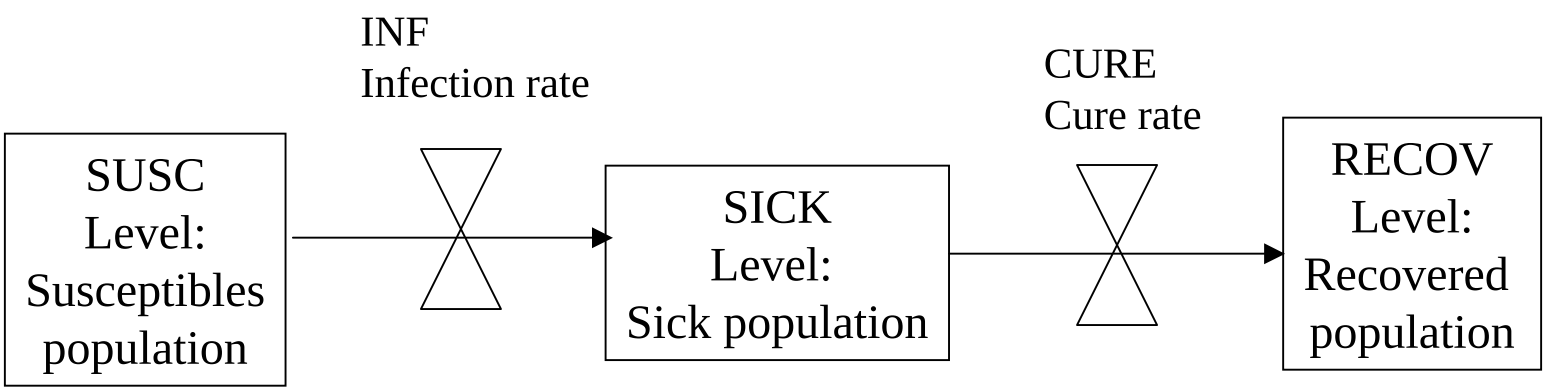
- Fig.(4.1) gives structure of an epidemic model (system).



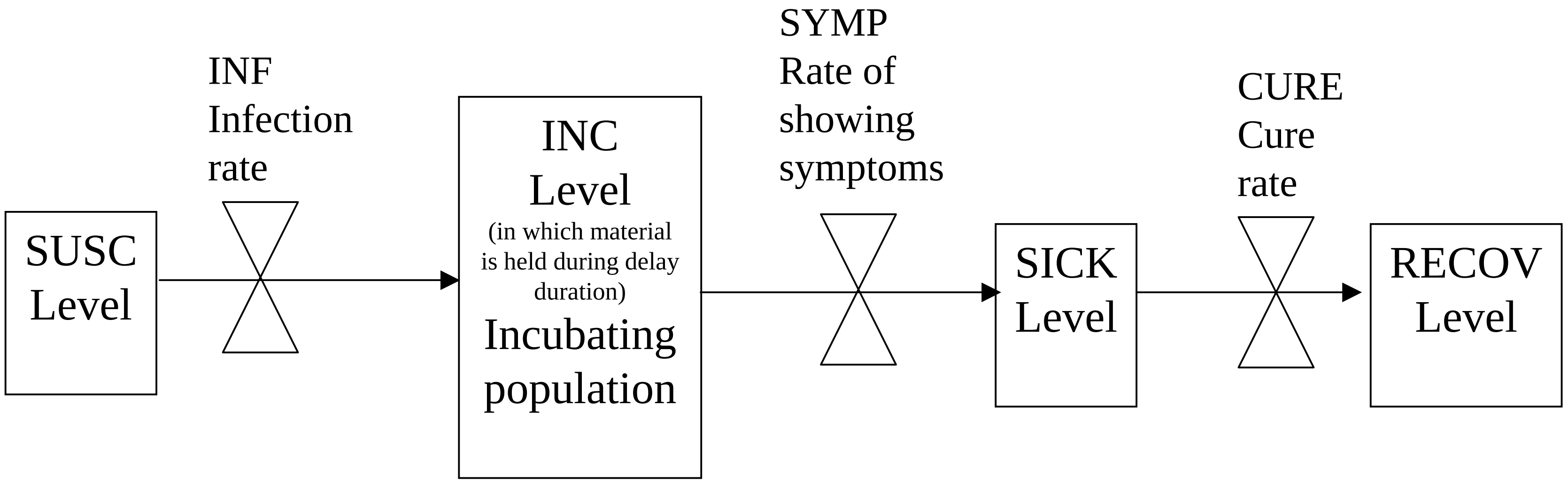
- Fig.(4.2) gives structure with incorporating delay present, of an epidemic model (system).



- Fig. (4.3) gives an epidemic model rate/level structure. (Note: Fig. (4.1) when drawn in DYNAMO (meaning dynamic models), which is a computer simulation language, gives Figure (4.3).)



- Fig. (4.4) gives the epidemic model rate/structure including the delay effect. (Note: Fig. (4.2) when drawn in DYNAMO (meaning dynamic models), which is a computer simulation language, gives Figure (4.4).)



- What mathematical equations will describe the delay effect?
- We will answer the question for the epidemic model rate/level structure in Fig. (4.1), i.e., Fig. (4.3).
- Specifically, need is to add a level, which stands for the effect of delay, to the epidemic model rate/level structure in Fig. (4.1), i.e., Fig. (4.3).

- This level is the population in which the disease is incubating and it is denoted as INC (i.e., incubating population).
- Any level is preceded by an inflow rate and succeed by an outflow rate. In this case,
  - Inflow rate (denoted by INRATE) = INF , and
  - Outflow rate (denoted by OUTRATE) = SYMP, where “SYMP” denotes “rate of showing symptoms”.

- Then for the system structure in Fig. (4.2), i.e., Fig. (4.4):
- *Equation for level "incubating population" (denoted by INC) representing level in which material is held during the delay duration (DEL)*:
  - INC.K= INC.J+DT*(INF.JK- SYMP.JK)
- *Equation for inflow into the level "incubating population"*:
  - INF*KL, which is exogenous to the delay
- *Steady state equation for the level*:
  - INC=INF*TSS
    - "TSS", i.e., "time to show symptoms" is a specific case of what can be generically described as the "delay duration" and can be in general terms denoted as "DEL".
- *Equation for outflow rate*: In steady state, outflow rate, i.e., "rate showing symptoms" (i.e., "SYMP") is equal to inflow rate, i.e., "infection rate" (i.e., "INF"). Hence from steady state equation for level we get:
  - SYMP.KL=(INC.K)/TSS

- Precisely above formulations are accomplished by the following single DYNAMO equation:
  - SYMP.KL=DELAY1(INF.JK,TSS)
- It is useful to note that “DELAY1” denotes a first order delay and the equation in fact is a first-order exponential material delay.

Microstructure of (Material) Delay-Generic Equations

- Generically, equations for the microstructure of system with delay then are as follows:
  - Let
    - LEV = Intermediate level introduced due to delay,
    - IN=Inflow rate to "LEV",
    - OUT=Outflow rate from "LEV",
    - DEL=Delay duration.

– Then,
  - Equation for intermediate level contributed by delay
    – LEV.K=LEV.J+DT*(IN.JK-OUT.JK)
  - Equation for inflow exogenous to delay
    – IN.KL
  - Steady state equation for level:
    – LEV=IN*DEL
  - Equation for Outflow rate:
    – OUT.KL=(LEV.K)/DEL

- The above is an EXPONENTIAL delay and it is termed as of FIRST ORDER because it contains one intermediate level.
- Following single equation in DYNAMO represent these first order delay formulations:
  - OUT.KL=DELAY1(IN.JK,DEL)

    where “DEL” denotes the “time for outflow to materialize”. (For example, in the example of epidemic model, “DEL” is “TSS”).

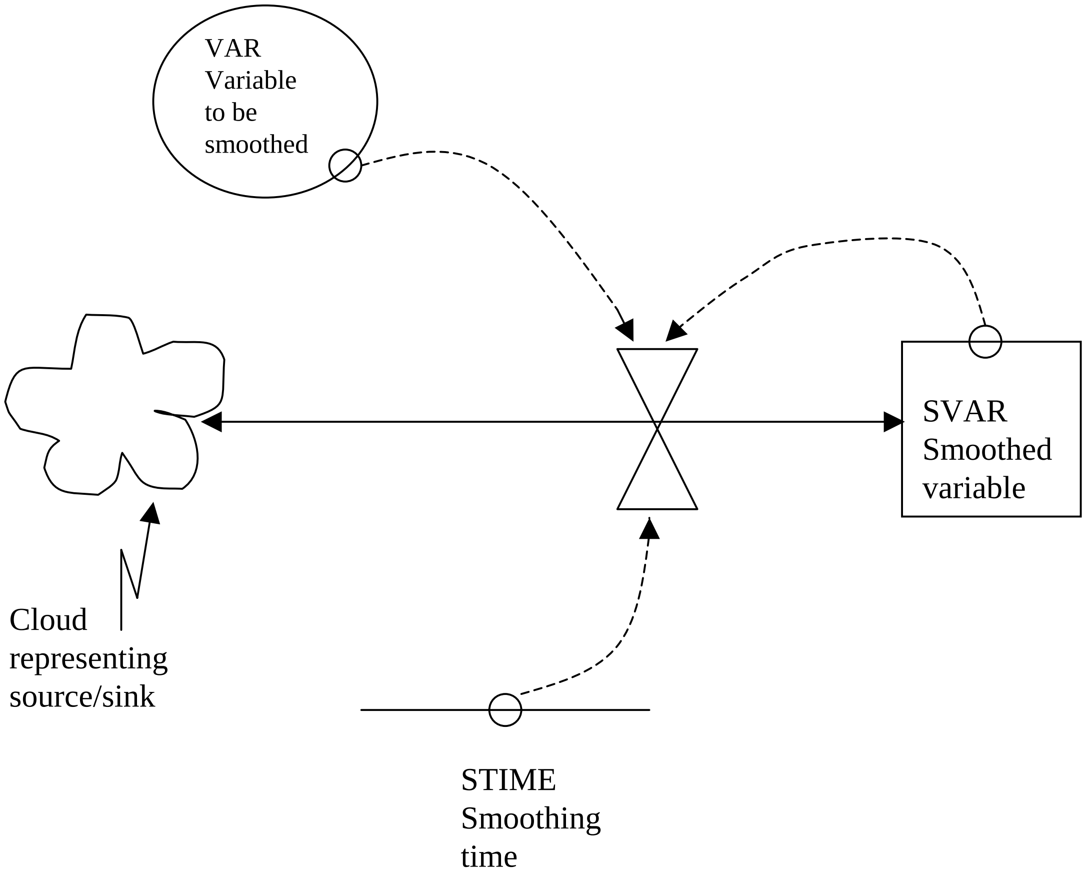
- Any number of first-order delays can be cascaded to produce a higher order delay. In practice, third-order delays are very common and will be denoted by DELAY3. For short it is written as:
  - OUT.KL=DELAY3(IN.JK,DEL)
  - in which the time postscript on IN and OUT should be noted. DEL is the total delay and each of the three first-order delays has a time-lag of DEL/3.
- Very high-order delays are approximated by cascading the required number of third-order delays.

Microstructure of (Information) Delay- "Smoothing" Information: the SMOOTH Function

- No corporate officer would interpret a day's jump in sales as a permanent trend on which to base decisions about inventories, production, or employment. She would want to "smooth" out randomness from sales data sufficiently to detect real trends. Sales would be averaged over some period of time.
- Because averaging and smoothing processes abound in dynamic systems, DYNAMO provides a function for the purpose, which is called "SMOOTH".

- Smoothing or averaging information requires that it be accumulated. An average is, after all, a sum, an accumulation.
- The DYNAMO formulation for a first order smooth or information average is equivalent to the structure shown in Figure (5).

Fig. (5): A First-order SMOOTH of a Variable



- Flow diagram in Fig. (5) is described by following equations:
  - L SVAR.K=SVAR.J+DT*SRATE.JK
  - N SVAR=VAR
  - R SRATE.KL=(VAR.K-SVAR.K)/STIME
- Or, more simply,
  - L SVAR.K=SVAR.J+(DT/STIME)(VAR.J-AVAR.J)
  - N SVAR=VAR
- Same thing is accomplished by the single auxiliary equation
  - A SVAR.K=SMOOTH(VAR.K,STIME)

- Alternate description
  - The well known exponential smoothing equation is:
    - At + (1-α) At-1
      - Where A is the average value of some variable C, t is time and α is a damping constant.
  - Rewriting,
    - At=At-1+α(Ct-At-1)
  - And if α=(DT)/(STIME) we have, in the notation of the conservation equation
    - A.K=A.J+(DT/STIME) (C.JL-A.J), where STIME is called the smoothing time.

Behavior of Delays

- In its steady state, a delay has no effect on the dynamics of the system because the outflow and the inflow rates are equal. If, however, the inflow rate varies, the outflow rate adjusts to the new inflow by following a pattern of Transient Behavior. Since the inflow rate is under its own controls, the transients in the output modify the control effectiveness and this accounts for the importance of delays in the dynamics of a system.

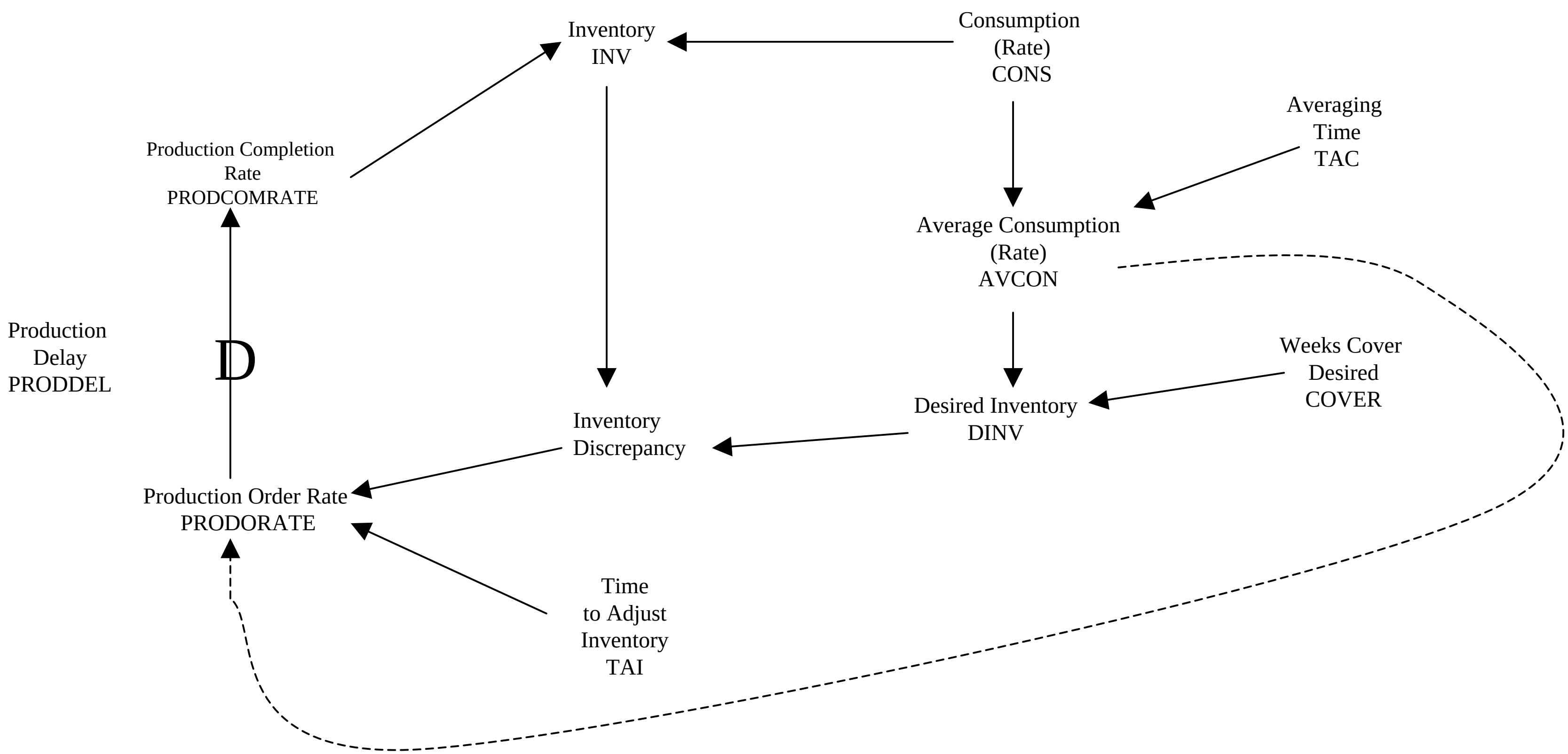
An Example: An Inventory Problem

- Having discussed dynamic processes and the basic components of system structure, one can apply these concepts to a simple example.
- The system in the example is a simplification of that used by many manufacturers of consumer goods, which are sold from stock.

- The stock is depleted by consumption which varies in two ways: firstly it moves fairly abruptly from one level to another; secondly even at a given level it still varies quite sharply from one week to another, as a result of all the myriad factors which affect purchasing decisions.
- It is assumed that it is not possible to tell in advance, or at the time, whether a movement in consumption is random noise or step change. The only way is to wait and see, and the consumption pattern cannot be forecast in any satisfactory way.

- The firm must decide the rate of production for stock, which involves a delay. The firm has to construct a policy, or decision- rule stipulating which information will be used in selecting the ordering rate, and how rapidly any gaps between actual and desired states are to be closed. The system structure is shown in Figure (6).

Fig. (6): A Simple Structure of Inventory System



- Choosing the Decision Rule of Policy to control (Production) Order Rate (i.e., PRODORATE or ORATE)
  - PRODORATE can be based on any number of factors.
    - 1. Inventory-based (IB) Rule:
      - The first is simply to consider the inventory gap and to have:
        - » PRDORATE=(DINV-INV)/TAI, and
        - » DINV=COVER×AVCON

- 2. Inventory and Order-based (IOB) Rule:
  - The second option is to take account of the average consumption, AVCON, as indicated by the dotted line, and write:
    - » PRODORATE(i.e., ORATE)= [(DINV-INV)/TAI]+AVCON
- Problem in the example is then analyzed to determine how each (and such other) rules would stand up to shocks imposed by the external environment.
- For this one may use test inputs, namely, a step, a sine wave, or ramp, etc. indicating change in consumption.

- Finally, it is useful to note that, for a given rule (i.e., for IB or IOB) and for a given test input, it is by varying parameter TAI that behavior of inventory is obtained. In other words, the optimum production order rate gets chosen in terms of the optimum parameter value “TAI”.

Influence Diagrams

- The basic tool for developing a dynamic model is the Influence Diagram.
- Figure (7) shows an influence diagram for the production-inventory control example.

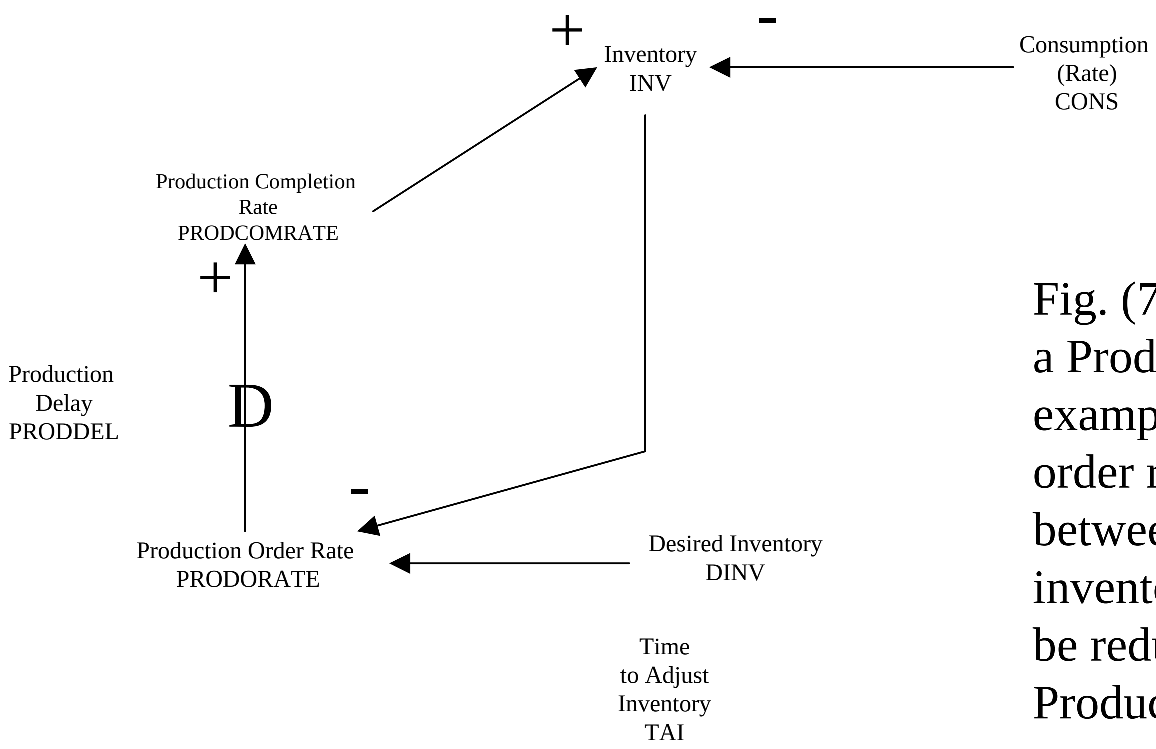


Fig. (7): Influence Diagram for a Production-Inventory control example, where production order rate is based on error between desired and actual inventory , with the error to be reduced over a fixed Production Adjustment Time.

- The influence diagram records the way in which system works. This is done by writing the names of the variables concerned and connecting them by an arrow, or influence line (or link). The direction of the arrow shows the direction of causation.

- The signs at the heads (points) of the arrows show the sign of the effects.
- The rule is: *if the head variable changes in the same direction as the tail variable, use a (+) sign, but if it changes in the opposite direction use a (–) sign; if the result is sometimes in the same direction and sometimes in the opposite direction use an asterisk.*

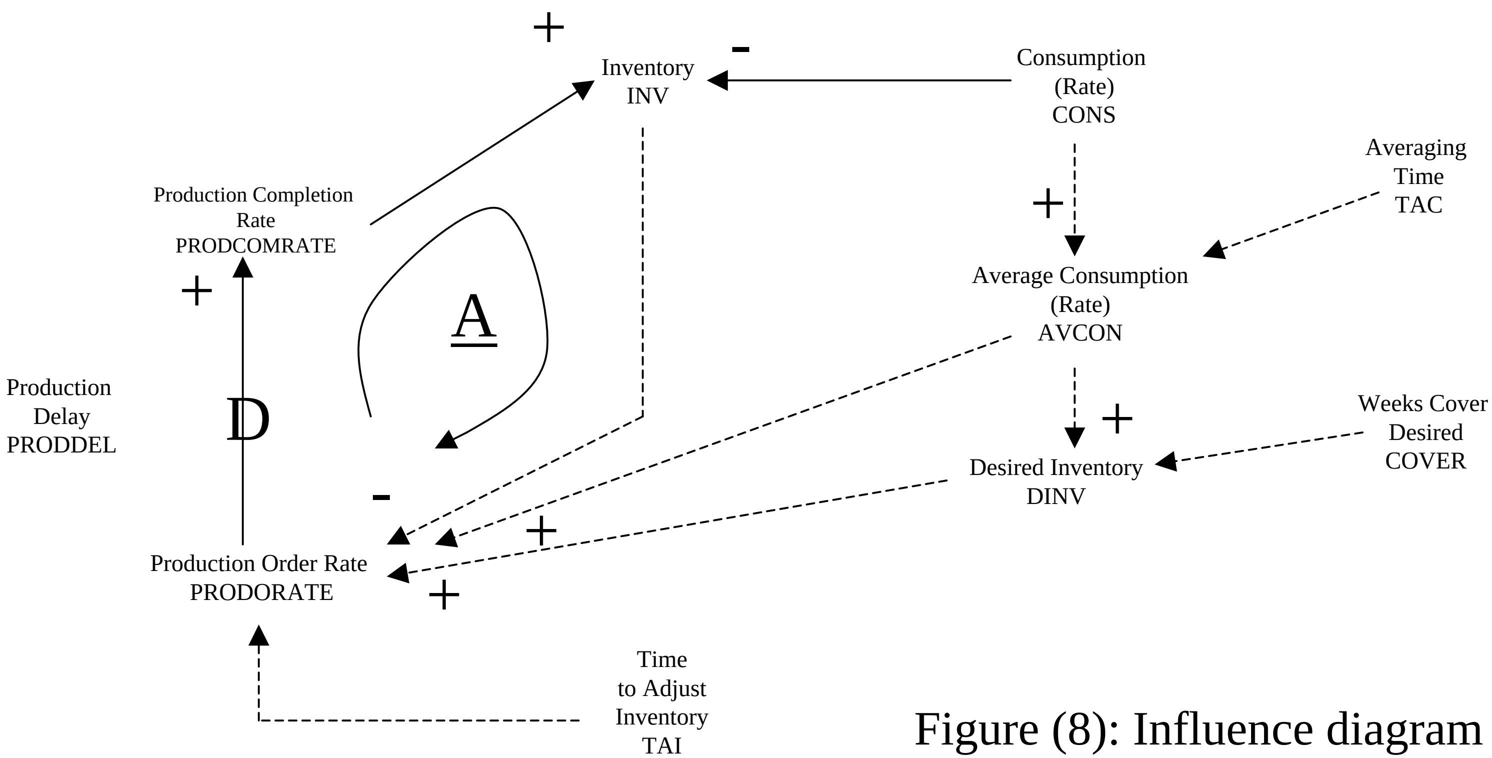


Figure (8): Influence diagram for the Production control problem, showing inclusion of more complicated control Rules, which take account of

Average Consumption

- The solid lines represent those flows, which are, in some sense, fixed and physical.
- The dotted lines, on the other hand, are flows of information, which management chooses to use in decision-making.

The Justification of Influence Diagram Links

- Conservation considerations
- Direct observation
- Instructions to that effect (commands)
- Accepted theory
- Hypothesis or assumption
- Statistical evidence
- Correlations (auto and cross)
- Enterprise knowledge management

Closure Test

- Influence diagram must pass a fundamental test in order to constitute a dynamic model, in that it must possess the property of CLOSURE.
- This means it must possess at least one feedback loop, and that all its variables lie on a loop, have been defined as exogenous inputs to to a loop, or provide supplementary output from a loop.

- The essential concept of a dynamic model is that its dynamics are produced by the operation of feedback loops in closed system (Refer Fig. (2) and Section “Significance of Feedback Loop” starting slide no. 5)
- Any model, which does not contain a feedback loop is, therefore a static model. Paradoxically, a static can produce dynamic behavior when it is driven by by an exogenous time-series (Fig. (1)), but this is simply due to the system state changing in response to the exogenous input and not under the influence of control policies based on the system state (which is what happens in a closed loop system as in Fig. (2)).

- As soon as a control law is introduced it must be based on some aspect of the system state; it makes the system a closed loop system, a feedback loop is immediately created and dynamic model comes into play.
- A test for closure then is: starting from any point in the influence diagram it must be possible to return to that point by following the influence lines, in the direction of causation, in such a way as not to cross one's track.

- This test applies to all points in the diagram, with certain exceptions, which are noted below.
  - Choose any arbitrary starting point and trace a path which returns to that point then a number of intermediate points will have been passed and, these, of course, lie on the feedback loop just traced out. Since they lie on this feedback loop they lie on a loop and, therefore, satisfy the closure test and can be dropped from further consideration.

- – Having taken one path in the diagram and proved that it is a feedback loop, one must apply the closure test to any remaining paths in the diagram to see whether they pass it, whether their variables are covered by one of the exceptions or whether, indeed, the system is not totally closed.
  - There are three situations in which it is permissible not to be able to return to the starting point without violating the rule of closure: parameter values, input variables, and supplementary variables.

Variables in Influence Diagram

- Level
- Rate
- Auxiliary variable
- Parameter value: These normally are constants, e.g, no. of weeks of average sales, which form target inventory.
- Input variable
- Supplementary variable: These are system performance indicators and play no part in the system or its control policies.

Fig.(9): Example

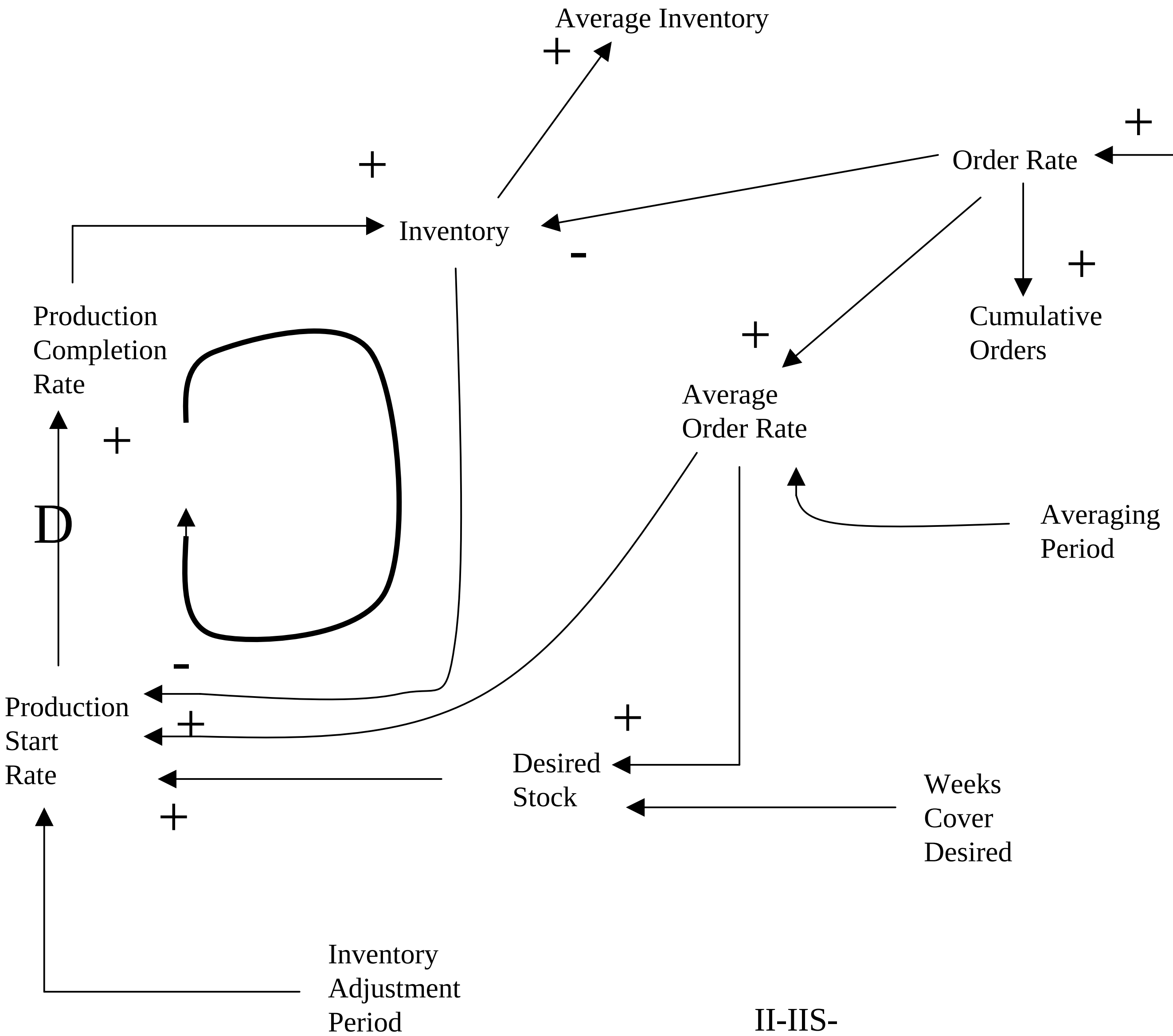


Figure (9): Example of a Production-Inventory System (Model) represented as Influence diagram including supplementary variables

- Influence diagram of the production-inventory control system in Figure (9) has:
  - One feedback loop (it is shown in the figure.)
  - Three parameters
    - Averaging period (for determining Average Order Rate)
    - Weeks Cover desired (for determining desired stock),
    - Inventory adjustment period (for determining Production Start Rate)
  - Two supplementary variables:
    - Average inventory,
    - Cumulative orders

Boundary

- The closure test shows when an influence diagram has become model.
- The concept of closure is very useful for another reason: the light that it sheds on the boundaries of the model and the derived notion of a hierarchy of increasingly complex models of a system.
- Closure test, thus, distinguishes between one model and a more complex model.

List Extension Method

- The technique which helps get started influence diagram, facilitates stopping it with the aid of closure test, and focuses attention on the purpose of the model is called the “List Extension Method”. (Cost-benefit Analysis of I*I is an analytical, scientific method, which goes beyond the List Extension method.)

Figure (10): Example of List Extension Method

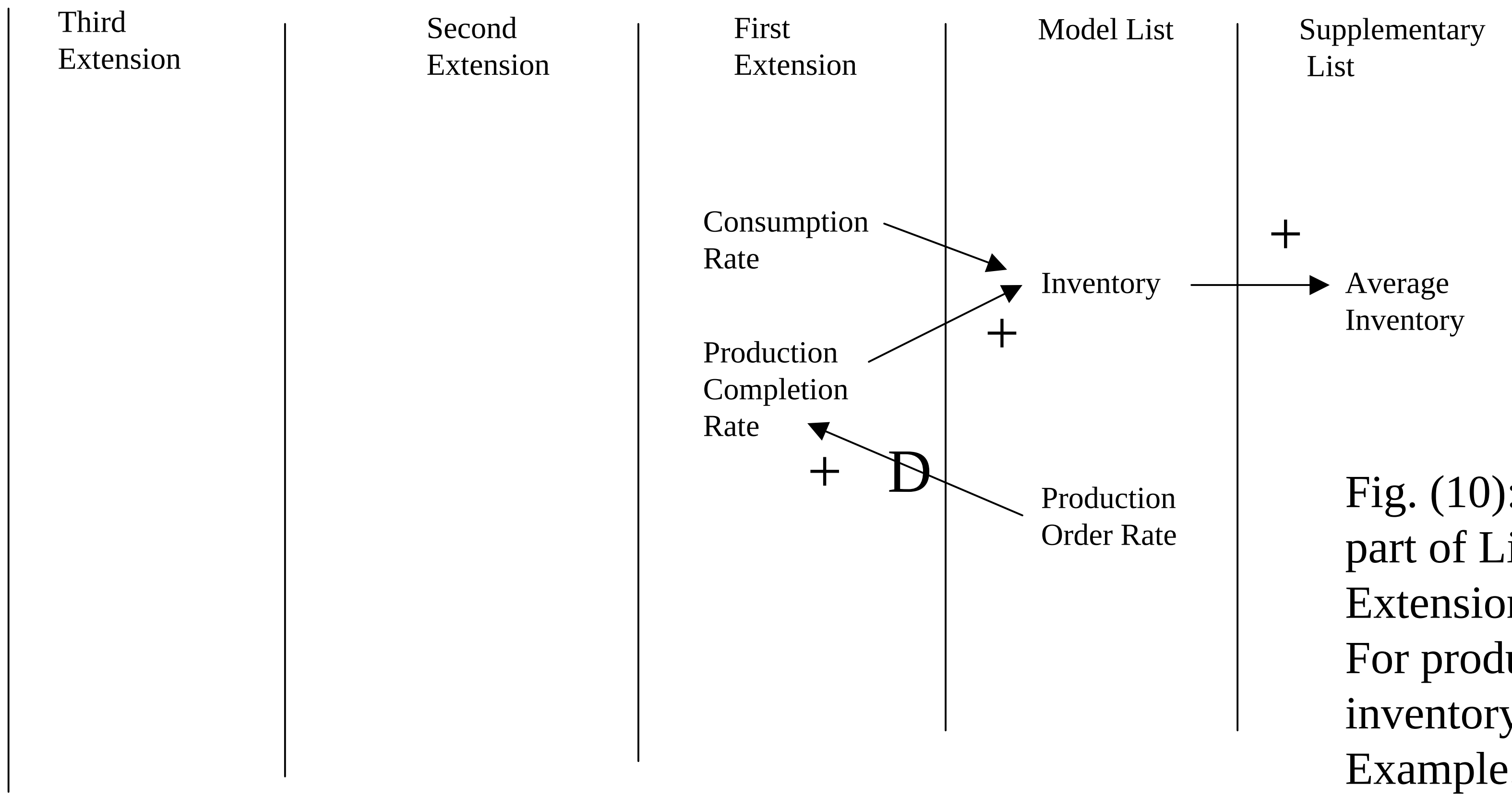


Fig. (10): First part of List Extension
For production-inventory Example.

Figure (11): Example of List Extension Method

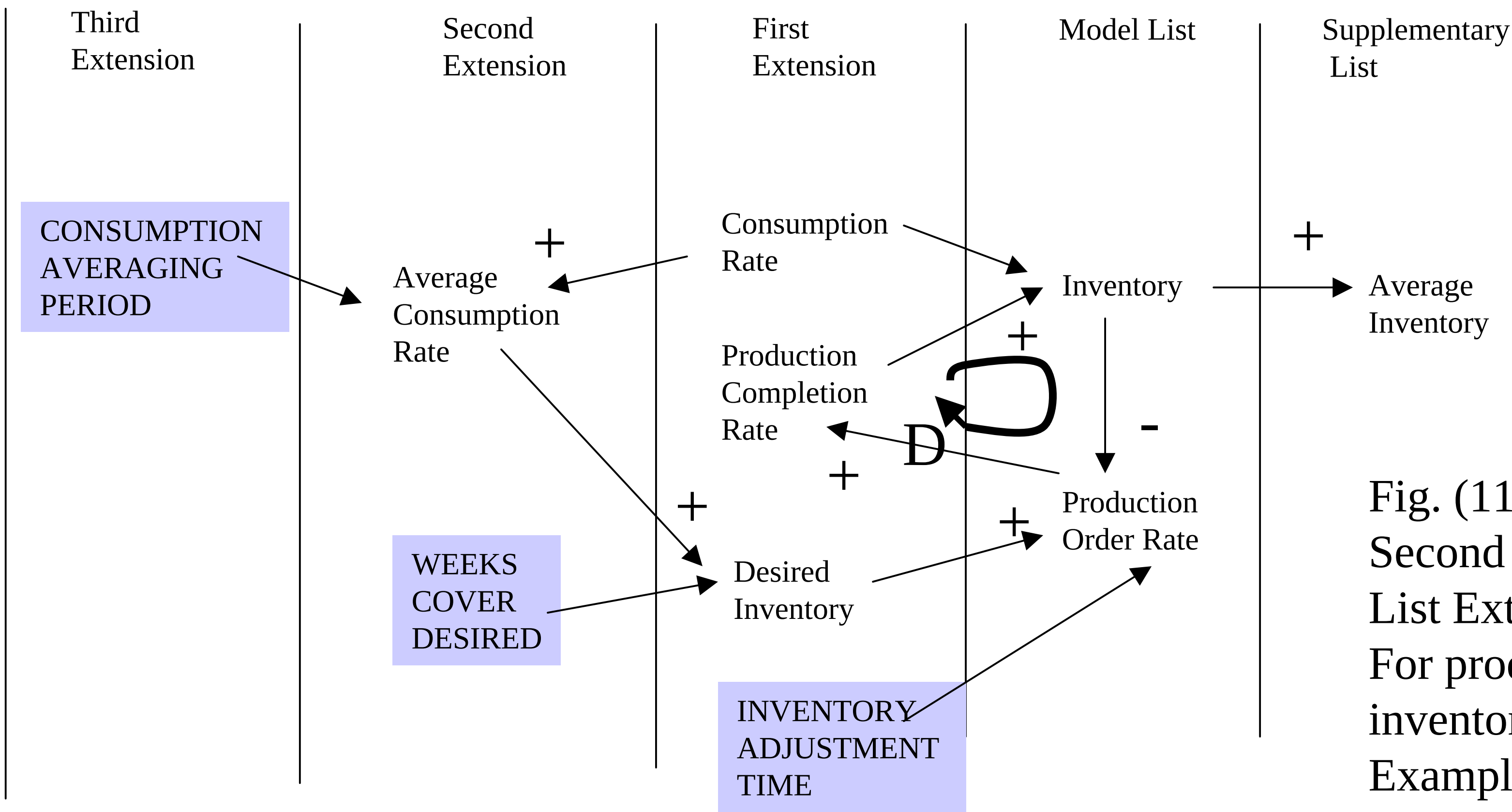


Fig. (11): Second part of List Extension For production-inventory Example.

List Extension Methodology explained with Examples (10) and (11)

- The purpose of the model is to explain the variations in inventory and production order rate and to enable the analyst to study the problem of devising improved control strategies.
- The first part of list extension is shown in Fig. (10).
- The Model List contains the names of the variables the behavior of which it is the purpose of the model to explain, or the control of which is aimed at.
  - It is good practice to limit the number of variables in the model list, in order to have reasonably clear and coherent purpose. (But opportunity requires maximal variables. Here comes in integrity optimization.)

- For each model variable one writes, in the first extension column, names of the variables, which most immediately affect it, drawing the influence lines.
  - Remember that variables in the model list may affect other variables in the same list, as may be the case for any of the lists, and variables in the earlier list may affect those in a later list.

- The lists must, therefore, e scanned for these connections as they are built up and the influence line drawn in.Even at this early stage attempt should be made to add signs to the links.

- After all the variables in the model list have been examined and the appropriate entries made in the first extension list, the resulting influence diagram is subjected to the closure test to see if an influence diagram, which is also a model has been produced.

- It it is, the next question is whether it is a good model. (I*I standard based on cost benefit analysis should answer the question.)
- It the influence diagram is not closed, the attention moves to the second extension list. This contains, for each variable in the first extension, which is not part of a feedback loop, which is not one of the exceptions to the closure rule, or which is not deemed to be modeled adequately (there are questions), the names of the variables, which most immediately affect it.

- Necessary links are drawn between the variables in the second extension and the variables in the first extension explaining, together with links denoting interconnections between variables in the second list,and between variables already entered in the first extension and the model list, and those being written into the second extension.
- When the second extension has been completed the closure test is again applied, and the process either terminates or continues.

Simple Example of List Extension

- To the right of the model list, the supplementary List contains Average Inventory, which is not part of the system, but is used as an indicator of system performance in order to discriminate between on control policy and another.
- The first extension included the consumption rate, which is an exogenous driving force, and the production completion rate.
- These links are justified on the basis of conservation considerations.The same considerations show that the production completion rate is affected with delay by the production order rate already entered in the model list.

- At this stage the analyst does not know what affects the production order rate.
- The production order rate could be determined in a host of different ways, but the analyst decides to find out what is done now.
- By the observation method, he discovers, say, that production order rate is determined by a comparison between actual inventory and its desired value and the latter is a fixed number of weeks of average consumption.

- **IMPORTANT**:
  - This number of weeks (Time), the consumption averaging time (Time), and the inventory adjustment time (Once again Time) (all of which are parameters) are all policy variables rather than technological variables (and they are therefore identified in capital letters in Fig. (11)).
  - Figure (11) also identifies the single feedback loop by broad line.

- The influence diagram in Fig. (11) passes the closure test and *hence a model is achieved*, but study of it will raise many questions (environmental anomalies?- Think) in the mind of analyst about how the system might be changed from its present form to a better one.

– For example:
  - Should the model parameters be fixed, possibly at values different from those in current use?, or
  - Should they be made into true variables dependent on other parts of the system, perhaps including quantities not yet in the diagram?, or
  - Should the consumption rate be used directly in determining production rate as well as indirectly?, or
  - Should desired inventory have non-linear dependence on average consumption rather than the present linear relationships?

- These and such other questions (e.g., is it more realistic to consider delay, which varies in time as activity level increases?) relate to the possibility of other control strategies and at the same time they can be resolved only through further information origination. Careful application of List Extension process accounts for this.

Thank You