System Dynamics Modeling and I*I Tech Development Revisited

II-IIS-MISCOULECT#Epilogue 3

12 APRIL 2007

………In preparation

References…

- These epilogue lectures have extensively referred to following literature:
  - R. G. Coyle, “Management System Dynamics”, A Wiley-Interscience Publication, John Wiley & Sons, NY, 1977,
  - George P. Richardson and Alexander L. Pugh III, “Introduction to System Dynamics Modeling with DYNAMO”, Productivity Press, Cambridge, Massachusetts, Boston, US, 1981,
  - CIIR Research papers,
  - Referrals on CIIR Web Site CIIR Systems”,

RECAP….. Fig.(9): Example

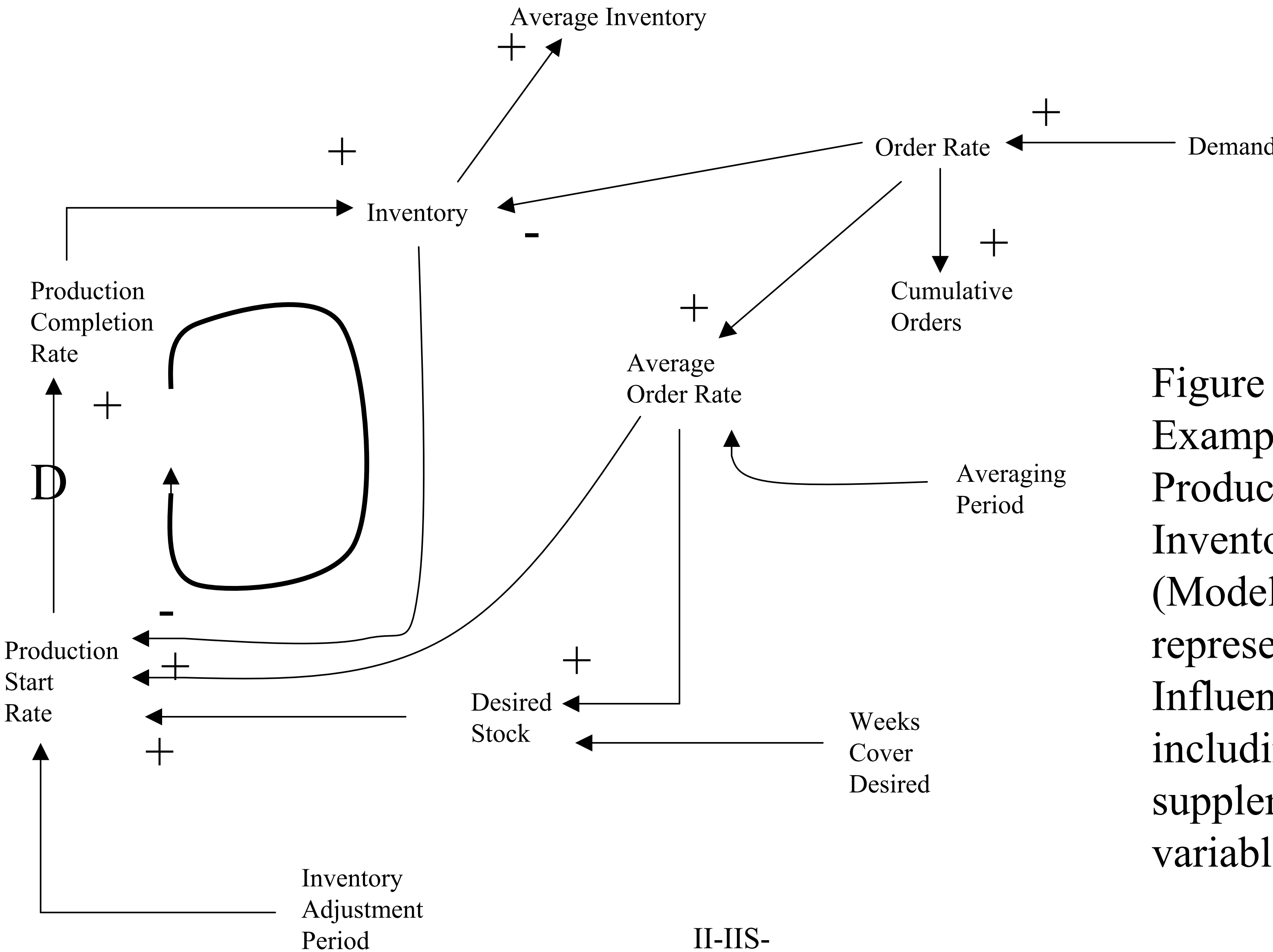


Figure (9): Example of a Production-Inventory System (Model) represented as Influence diagram including supplementary variables

Epilogue 2 Lecture Starts

Mathematical equations in System Dynamics – Preliminary observations on Level Equations

- A s discussed, level variables are the means by which the system acts to control itself through its decision rules, which produce the flows or rates.
- Specifically, a variable that accumulates over time an inflow and/or an outflow is a level variable.

- There are two kinds of level equations:
  - the conserved level in which the flows into the level are, by their nature, indestructible, e.g., product, cash, orders, etc; and
  - the smoothed level, which operates only on information and where the requirements of conservation do not apply.

- The basic form of the conserved level equation is:
  - LEV.K=LEV.J+DT x ($\pm$ RATE1.JK$\pm$RATE2.JK$\pm$…..)
- The other basic permissible form of a level equation is:
  - LEV.K=LEV.J+(DT/ATC)x(RATE.JK-LEV.J)
    - Where RATE is an input rate,
    - LEV is a level which is being smoothed, or average value of rate,
    - ATC is an adjustment time constant called the Smoothing
    - *Note (1): The second bracketed term must contain a rate and the level which appears on the LHS of the equality.*
    - *Note (2)* : A smoothed level always has the same dimensions as its input rate.

Mathematical equations in System Dynamics – Preliminary observations on Rate Equations

- Rate equations in a system dynamics model translate planning and system pressures into actions altering state of the system.
- Various types of RATE equations are:
  - CONST x LEVEL.K
  - LEVEL.K/LIFE
  - (GOAL.K-LEVEL.K)/ADJTM
  - AUX.K x LEVEL x K and LEVEL.K/AUX.K
  - NORM.K + EFFECT.K
  - NORM.K x EFFECT.K

Mathematical equations in System Dynamics – Preliminary observations on Auxiliary Equations

- A variable modeled as an auxiliary in a computer simulation in System Dynamics methodology represents *information* in the system.
- Since all information eventually traces back to the system levels, it is conceivable that one could write a model without using auxiliary equations, formulating all rate equations solely in terms of levels and constants.
- However, the resulting model listing would probably be nearly unreadable and un-tractable to the simulator.
- Consequently, one captures some pieces of system information in auxiliary variables., which aid capturing the system complexity and formulating rate equations.

Deriving Mathematical Equations for Problem System Structure

- In Epilogue 2 the Question dealt with is how to construct mathematical equations for a (causal loop based) problem system structure (model), which may be represented by "inference diagram" or "flow models".
- We first take the case of system structure (model) represented by influence diagram based on causal loops.

Example of Production-Inventory System where Decision Rule is to control Production Start Rate using inventory gap

- Consider a simplified production-inventory system as derived from example in Figure (9) of Lecture II-IIS-MISCOULECEPILOGUE2R21MARCH07.
- The simplified system is given in Figure (12).
- Note how delay in Figure (9) is represented here.

Fig.(12): Example

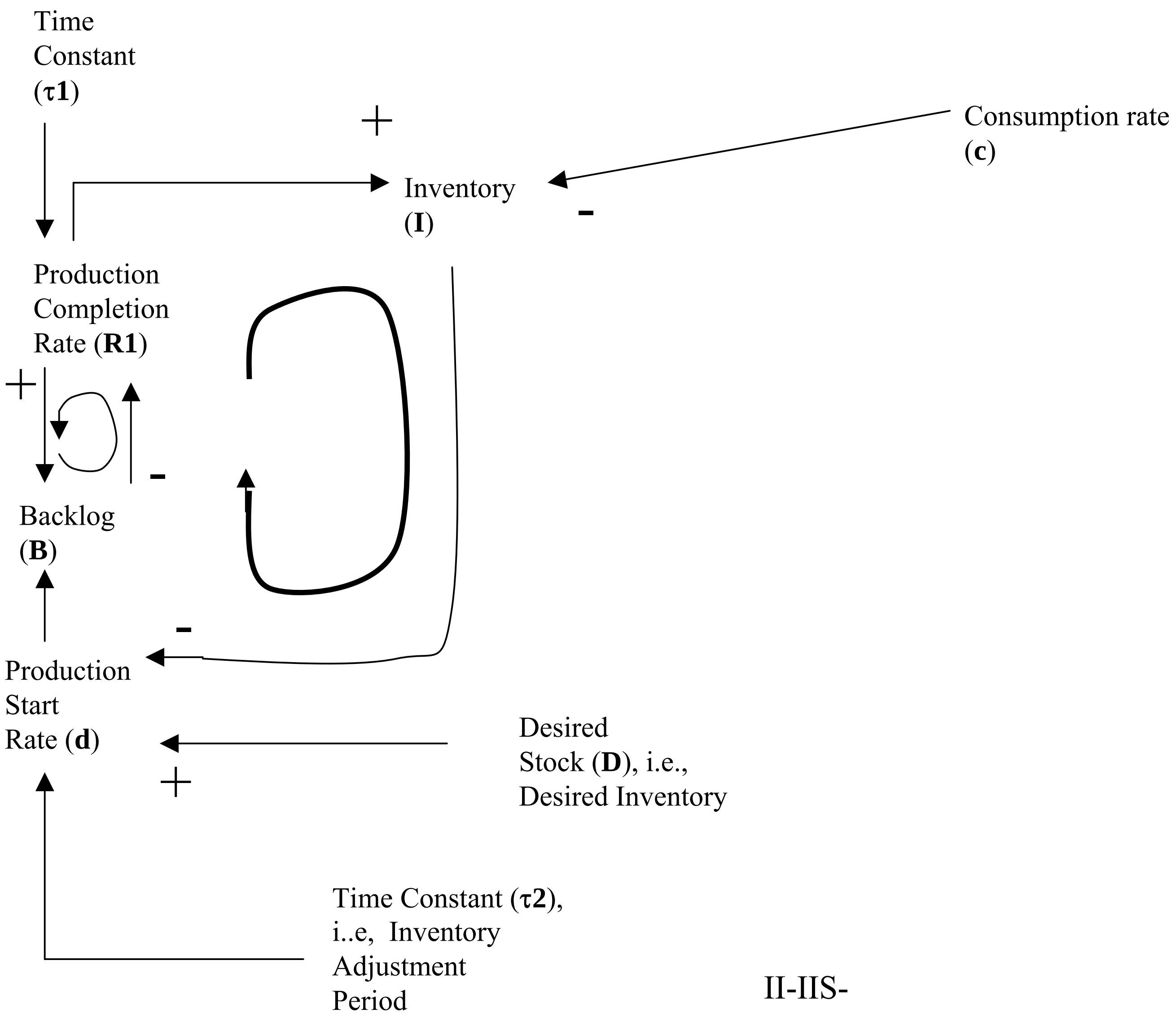


Figure (12): Example of a Simplified Production-Inventory System (Model) represented as Influence diagram

Figure (13): List Extension representation for example in Figure (12)

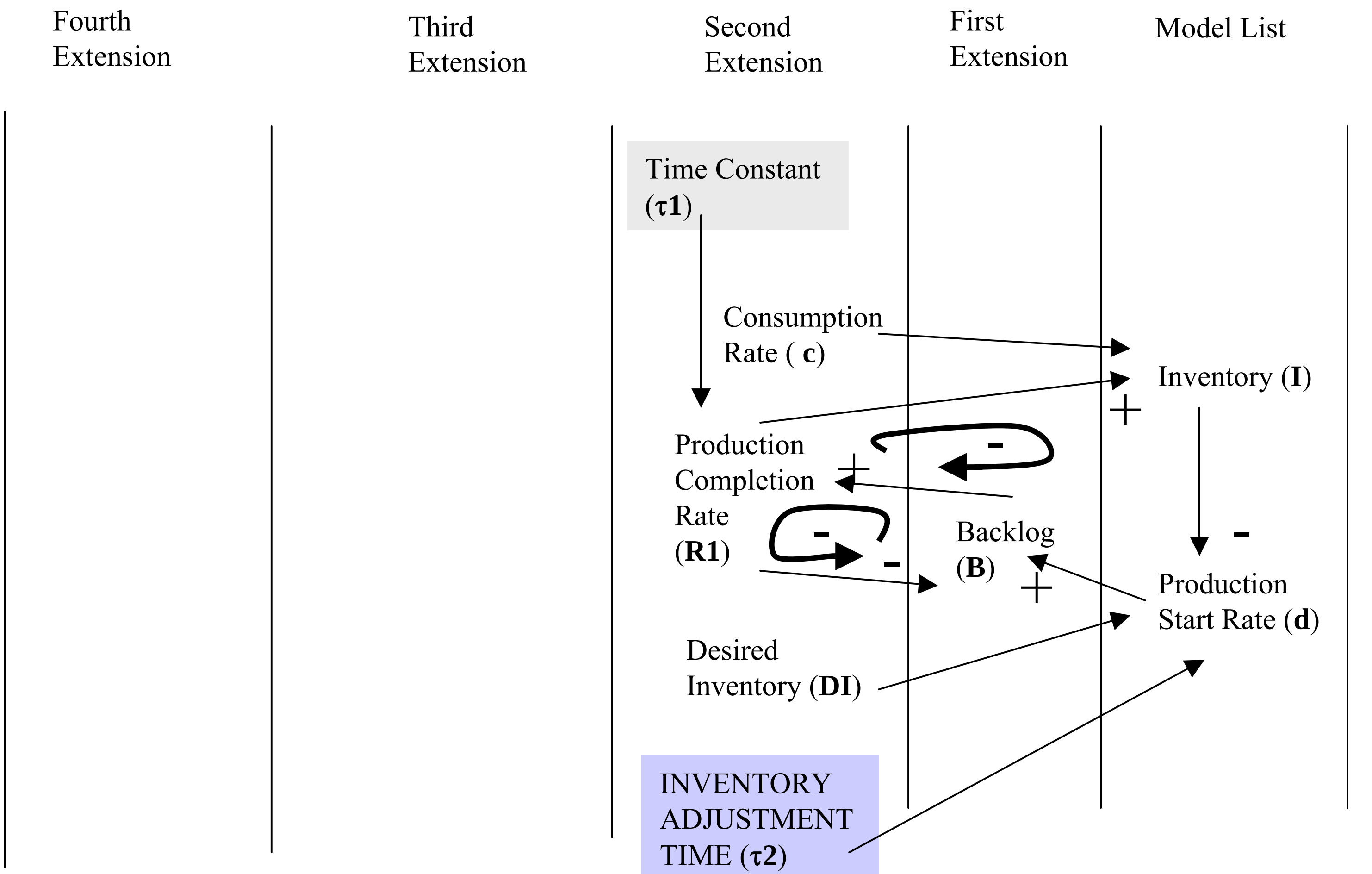


Fig. (13): List Extension For production-inventory system structure in Figure (12) Example.

Mathematical equations for Example in Figure (13)

- Note that the production-inventory system structure in example in Fig. (13) has production start rate “d” linked to an exogenously determined consumption rate “c”.
- What is controlled?
  - “B” represents the lag in production process and “I” is inventory, which is controlled to a desired value “DI”.

- Implications:
  - When inventory "I" is equal to desired inventory "DI", production start rate "d" is equal to consumption rate "c".
  - Further, decision rule, i.e., control rule here is simply based on (inventory) error gap. We call this *Rule 1*.

- The basic equations are:
  - R1= (B)/(τ1) …………… Equation (1)
  - İ = I̲ = D(I) = I̲ = (R1) – (c) … Equation (2)
  - D(B) = (d) – (R1) ………… Equation (3)
  - (d) = [(DI) – (I)]/(τ2) …… Equation (4)
  - (d) = (e)/ τ2 …………….. Equation (4.1)
  - ……… Equation (4)
- Putting value for R1 from (3) into (2), we get:
  - [D(I)] = {(d) – [D(B)] – (c)] ……….. (5)

- Further recognizing that when “I” equals “DI”, “d” equals “c”, by differentiating in both sides in Equation (2), we get:
  - $[D(R1)] = [D^2 (I) – D(d)]$ ……. (6)
- And by differentiating both sides in Equation (1) and putting value of D(R1) from (6), we get:
  - $[D(B)] = [\tau 1\ D(R1)]=\tau 1[D^2 (I) + D(d)]$ … (7)

- Now from Equations (5) and (7) we get:
  - $D(B) = \tau 1[D^2 (I) + D(d)] = [(d) - D(I) - (c)]$ ..(8)
- Now, from (4) we have,
  - $(I) = \{(DI) - [(\tau 2) \times (d)]\}$ ……….. (9)
- Hence, given that DI and τ2 are constants, we have:
  - $D(I) = -[(\tau 2) \times D(d)]$ …… (10)
  - $D^2(I) = -[(\tau 2) \times D^2(d)]$ ……(11)

- Using (10) in (8), we get:
  - $\tau 1\{-[(\tau 2) \times D^2(d)] + D(d)\} = [(d) + [(\tau 2) \times D(d)] – (c)]$,
  - $- \{(\tau 1) (\tau 2) D^2(d)\} – \{(\tau 1) D(d)\} = [(d) + [(\tau 2) \times D(d)] – (c)]$,
  - $- [(\tau 1) (\tau 2) D^2(d)] – [(\tau 2) \times D(d)] – [(d)] = - (c) - [(\tau 1) \times D(d)]$ ………………… (12)
- Recognizing "d" equals "c" when "I" equals "DI" and multiplying by (-1) on both sides in Equation (12), one gets:
  - $[(\tau 1) (\tau 2) D^2(d)]+[(\tau 2) D(d)]+[(d)] = (c)+[(\tau 1) D(c)]$ .. (13)

- Laplace transformation of Equation (13) with zero initial conditions leads to:

$$\frac{D(s)}{C(S)} = \frac{[\ 1 + (\tau 1)\ s]}{[\ (\tau 1\ \tau 2)\ s^2 + (\tau 2)\ s + 1]} \quad \ldots (14)$$

- Now one may write the computer program for these equations in DYSMP, DYNAMO, or Vensim.

Example of Production-Inventory System where Decision Rule is to control Production Start Rate by equaling it to average consumption, plus stock adjustment factor

- Consider a (more complex than in Figure (12)) production-inventory system as derived from example in Figure (9) of Lecture II-IIS-MISCOULECEPILOGUE2R21MARCH07.
- The system structure is given in Figure (14).

Fig.(14): Example

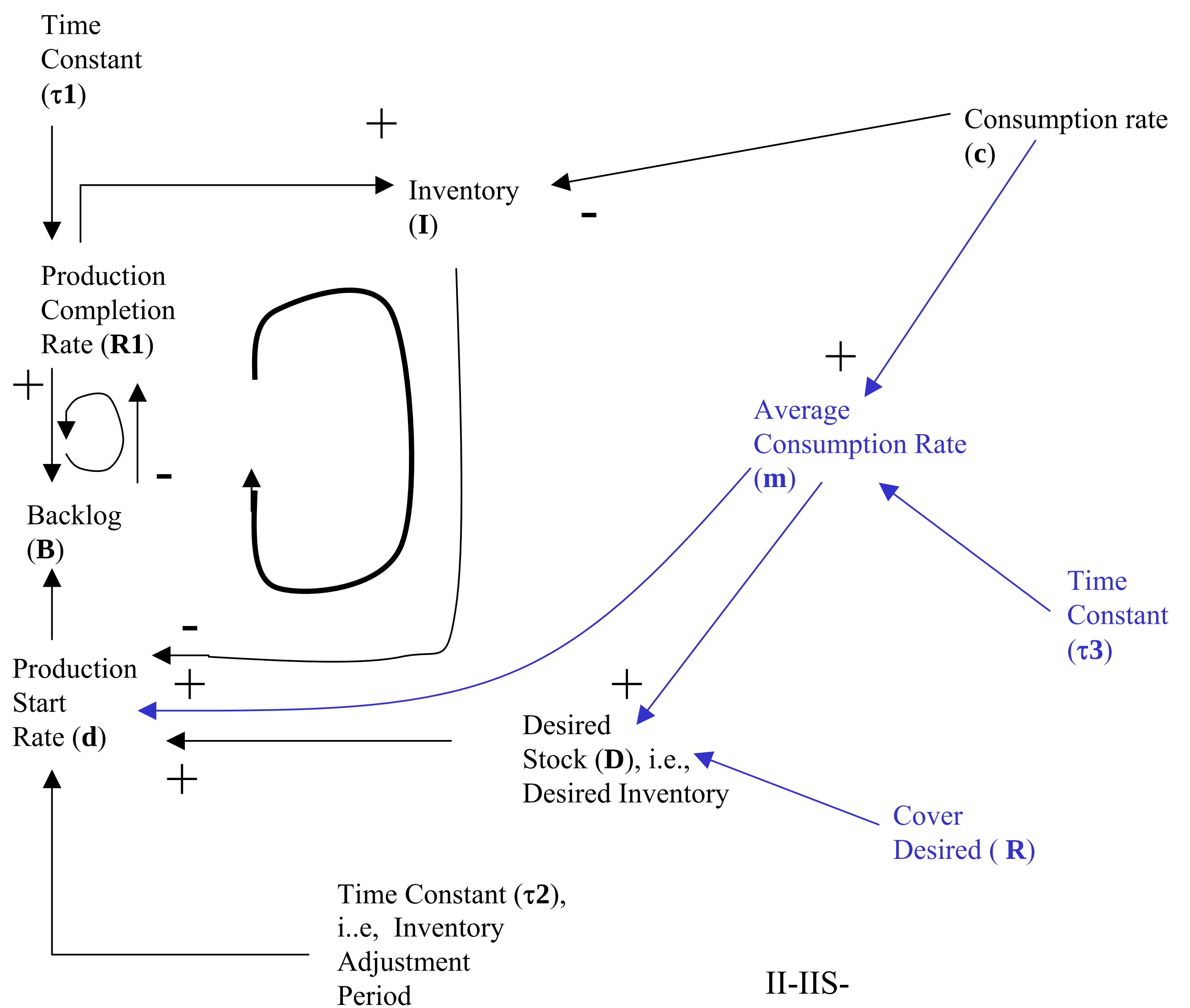


Figure (14): Example of a (more complex) Production-Inventory System (Model) represented as Influence diagram

Figure (15): List Extension representation for example in Figure (14)

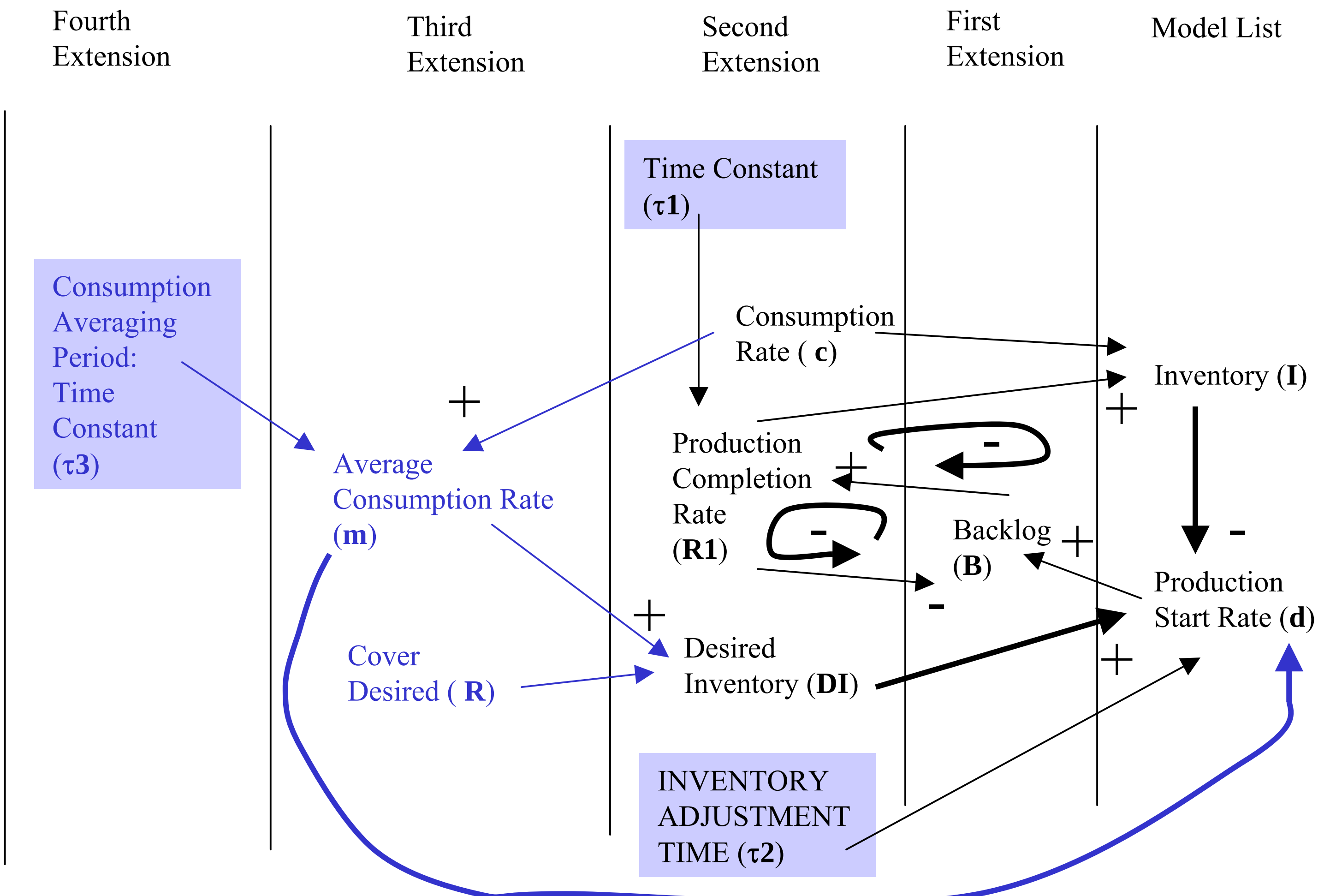


Fig. (15): List Extension For production-inventory system structure in Figure (14) Example.

Deriving Mathematical Equations for Production-Inventory Problem System Structure in Figure (15)

- Development of mathematical equation for the model of environmental dependency for the Production-Inventory System problem modeled as dynamic system
- Let notation D denote the first derivative operator d/dt, notation $\underline{D}^{2}$ the second derivative d^2/dt^2 operator and so on.
- Then, notation, $\underline{D}^{n}$ would denote nth derivative d^n/dt^n.

- Establishment of the additional information origination requirements for improving I*I of the Production-Inventory System problem by modeling it as a dynamic system brings the problem discussion to its last leg, which is developing mathematical equations for the corresponding I*I Process Level I.

- If we are taking the static model of the problem, the mathematical equation for the inventory rate is given by:
-
- $$\frac{dI}{dt} = \dot{I} = D(I) = \underline{I} = p - c,$$
- where p is production rate, c consumption rate and I inventory.
- Due to the environmental-variations-knowledge factors as in Figure (14), it is this equation that would change. What will that equation be when for simplicity, say, by controlling the variable “R” that improvement in integrity is sought?

- **Derivation of Transfer Function $\{G_2(s)=[I(s) \div C(s)]\}$, which is controlled by "R"**
- Need is to develop a mathematical equation for transfer function $\{G_2(s)=[I(s)/ C(s)]\}$, which is controlled by "R". Here "s" denotes the Laplace variable and I(s) is the Laplace Transform of inventory variable I(t) ands C(s) of consumption rate variable c(t).
- Within this framework, for the above Production-Inventory System, the basic system equations work out to be as follows.

- **Basic system equations**

- For the Production-Inventory System modeled as dynamic equation, the basic system equations work out to be as follows:

- $D = R \times m$ ……. Equation (1).

- $\underline{D}$ (m) = $\{[c-m]\div(\tau 3)\}$ ………… Equation (2).

- $O = [\{(D-I)\div(\tau 2)\} + m]$ ……….. Equation (3).

- $p = (B\div\tau 1)$ ……… Equation (4)

- $\underline{D}$ (B) = $(O - p)$ ……. Equation (5)

- $\underline{D}$ (I) = $(p-c)$ ………… Equation (6)

- **Solving the basic system equations for Transfer Function derivation**

- Solving for production in terms of consumption we have:

- $D = \{I + (\tau 2 \times O) - (\tau 2 \times m)\}$ ……………. Equation (7)

- $= \{I + (\tau 2 \times O) - (\tau 2 \times (D \div R)\}$ ……… Equation (8)

- $\therefore D[1+(\tau 2 \div R)] = \{I + (\tau 2) \times (O)\}$ ………Equation (9)

- Differentiating on both sides of equation (9), we get equation (10),

- $\{\underline{D}\,(D)\,[1+(\tau 2 \div R)]\} = \{\underline{D}\,(I) + \tau 2\,\underline{D}\,(O)\}$ ……………… Equation (10)

- Now, from Equation (5) we get,

- O = { D (B) + p} ………. Equation (11)

- But from Equation (4), we get,

- B = {(τ1) x (p)} …….. Equation (12)

- ∴ D (B) = {τ1 x D (p)} …… Equation (13)

- Using equation (13) in equation (11) gives, equation (14):
-
  - O = (τ1 x D (p) + p} ………. Equation (14)
  -
  - = (τ1 D + 1) p ……………Equation (15)
-
- We note that Equation (10) has in it quantity D (O). Therefore, by differentiating on both sides of Equation (15), we get value of (D O) as given by Equation (16):
-
  - D O = [τ1 D ^{2} + D] p …….. Equation (16)
-
- Now using Equations (6) and (16) in Equation (10) gives Equation (17) as follows,

- $\underline{D}$ D [1+(τ2÷R)] = {p – c} + {(τ2) [τ1 $\underline{D}^{2}$ + $\underline{D}$] p} …….. Equation (17)
-
- Differentiating on both sides of Equation (17) we get Equation (18),
-
- $\underline{D}^{2}$ D [1+(τ2÷R)] ={$\underline{D}$ (p) – $\underline{D}$(c)} + {(τ2) [τ1 $\underline{D}^{3}$ + $\underline{D}^{2}$] p} …….. Equation (18)
-
- Also by rearranging the terms in Equation (2) and differentiating on both sides gives Equation (19),
-
- (τ3) $\underline{D}^{2}$ (m) = [$\underline{D}$ (c)- $\underline{D}$ (m)] ………… Equation (19).
-
- But from Equation (1), we have $\underline{D}$ (m) = {$\underline{D}$ (D) ÷ R}. Substituting for $\underline{D}$ (m) in Equation (19) gives Equation (20),
-
- (τ3) {$\underline{D}^{2}$ (D) ÷ R} = [$\underline{D}$ (c)- { $\underline{D}$ (D)÷R}] ………… Equation (20).
-

- Substituting in Equation (20) for $\underline{D}^2$ (D) from Equation (18) and for $\underline{D}$(D) from Equation (17) gives Equation (21),
-
- $[\tau 3/R] \times [1/(1+\tau 2/R)] \times [\underline{D}(p)-\underline{D}(c)+\tau 1\tau 2\underline{D}^2(p)+\tau 2\underline{D}^2(p)] =$
- $= \underline{D}(c) - [(1/R) \times [1/(1+\tau 2/R)] \times [p-c+\tau 1\tau 2\underline{D}^2(p)+\tau 2\underline{D}(p)]$ …….Equation (21)
-
- Rearranging the terms and taking Laplace Transform with zero initial conditions leads to the system transfer function G(s) in Equation (22):
-
- $$G_1(s) = \frac{P(s)}{C(s)} = \frac{1+(R+\tau 2+\tau 3)s}{1+(\tau 2+\tau 3)s+\tau 2(\tau 1+\tau 3)s^2+\tau 1\tau 2\tau 3s^3}$$ ………Equation (22)
-

- Carrying out further operations on the numerator and denominator in Equation (22) gives Equation (23).

$$\frac{P(s) - C(s)}{C(s)} = \frac{[1+Rs+\tau 2s+\tau 3s] - [1+\tau 2s+\tau 3s+\tau 2(\tau 1+\tau 3)s^2+\tau 1\tau 2\tau 3s^3]}{1+(\tau 2+\tau 3)s+\tau 2(\tau 1+\tau 3)s^2+\tau 1\tau 2\tau 3s^3]}$$

$$= \frac{s\,[R - \tau 2(\tau 1+\tau 3)s + \tau 1\tau 2\tau 3 s^2]}{[1+(\tau 2+\tau 3)s+\tau 2(\tau 1+\tau 3)s^2+\tau 1\tau 2\tau 3 s^3]} \quad \text{……Equation (23)}$$

- However taking Laplace Transform on both sides of Equation (6), we get Equation (24).

  - $s\,I(s) = P(s) - C\,(s)$ ………….. Equation (24)

- Substituting RHS of Equation (24) in LHS of Equation (23) and rearranging the terms gives Equation (25).

$$I(s) = \frac{[R - \tau 2(\tau 1+\tau 3)s - \tau 1\tau 2\tau 3s^2]}{[1+(\tau 2+\tau 3)s+\tau 2(\tau 1+\tau 3)s^2+\tau 1\tau 2\tau 3s^3]} C(s) = G_2(s)\, C(s) \quad \text{……..Equation (25)}$$

- Equations (22) and (25) give in different forms the transfer function for the Production-Inventory System problem modeled as dynamic system, the convenient one can be chosen for further analysis.

- **Derivation of $G_2(s)$ in State Space Matrix Form**
- It is useful to transform $G_2(s)$ in state space matrix form by developing the state space model of the dynamic system of the Production-Inventory problem. State space model is useful as it is amenable to time-domain analysis by development of computer application.
-
- For this, let us define three state variables as in Equation (26):
-
- x1=B, x2=m, x3=I …………………………. Equation (26)
-

- Regrouping the Basic System Equations from (1)-(6) so as to involve only variables B, m, I and variable c, we get Equations (27)-(29).
-
- $\underline{D}\,(B) = \left[\begin{array}{ll} (R + \tau 2)\, m - I & B \\ & \\ \tau 2 & \tau 1 \end{array} \right]$
- -
- …………. Equation (27)

- $\underline{D}$ (m) = {[c-m]÷(τ3)} ……….. Equation (28) {Note it is same as Equation (2)}
-
- $\underline{D}$ (I) = [B/τ1] – c ………. Equation (29)

- This leads to the State Space representation of the Production-Inventory System problem modeled as dynamic system and the same is given by Equations (30) and (31).

- Given that $\underline{X} = [x1 \quad x2 \quad x3]^T$, then

$$\frac{d\underline{X}}{dt} = \begin{bmatrix} -(1/\tau 1) & [(R+\tau 2)/\tau 2] & -(1/\tau 2) \\ 0 & -(1/\tau 3) & 0 \\ (1/\tau 1) & 0 & 0 \end{bmatrix} \underline{X} + \begin{bmatrix} 0 \\ (1/\tau 3) \\ -1 \end{bmatrix} c \, , \quad \text{…Equation (30)}$$

$$y = [\,0 \quad 0 \quad 1\,] \; X \, , \quad \text{…… Equation (31)}$$

where y gives x3, which is noting but inventory I(t).

- That is the matrix Equations (30) and (31) can be presented as Equations (32) and (33), respectively.
-
- $\underline{D}\,(\underline{X}) = \underline{A}\ \underline{X} + \underline{M}\ c$, ……. Equation (32)
-
- $y = \underline{H}\ \underline{X}$, ………. Equation (33)
-
- where state transition matrix, input matrix, and observation matrix H are defined with values obtained form Equations (30) and (31). Needless to say there would be distortion and noise factors impacting these equations, too.

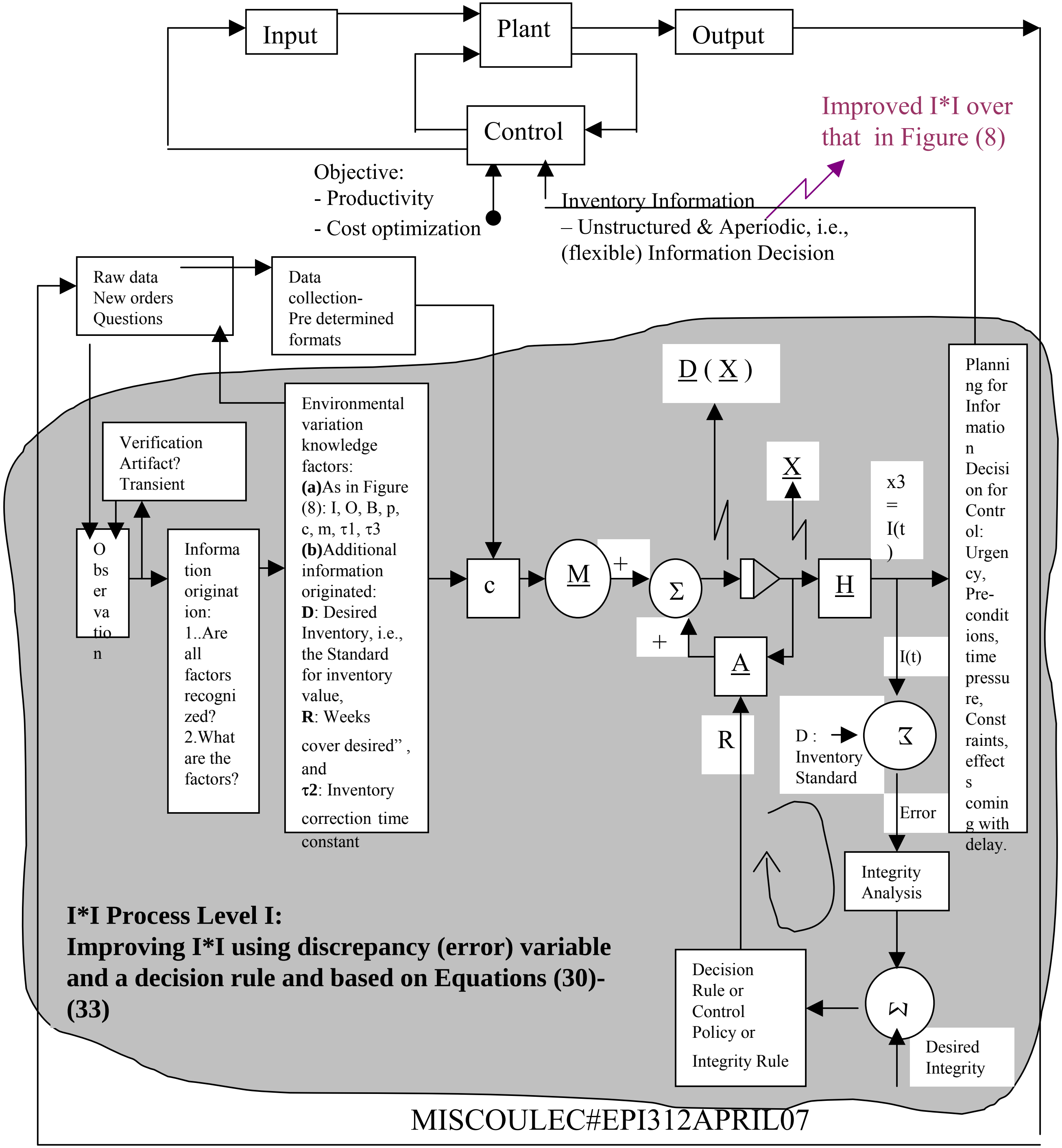


Figure (10): System's model of I*I Process Level I resulting in improvement of I*I using discrepancy (error) and a decision rule for the Production-Inventory System problem represented vide the dynamic systems representation in Section (2.8.5).

- Now one may write the computer program for these equations in DYSMP, DYNAMO, or Vensim.

Causal-loop diagrams leading to Development of Flow Diagrams

Business Model

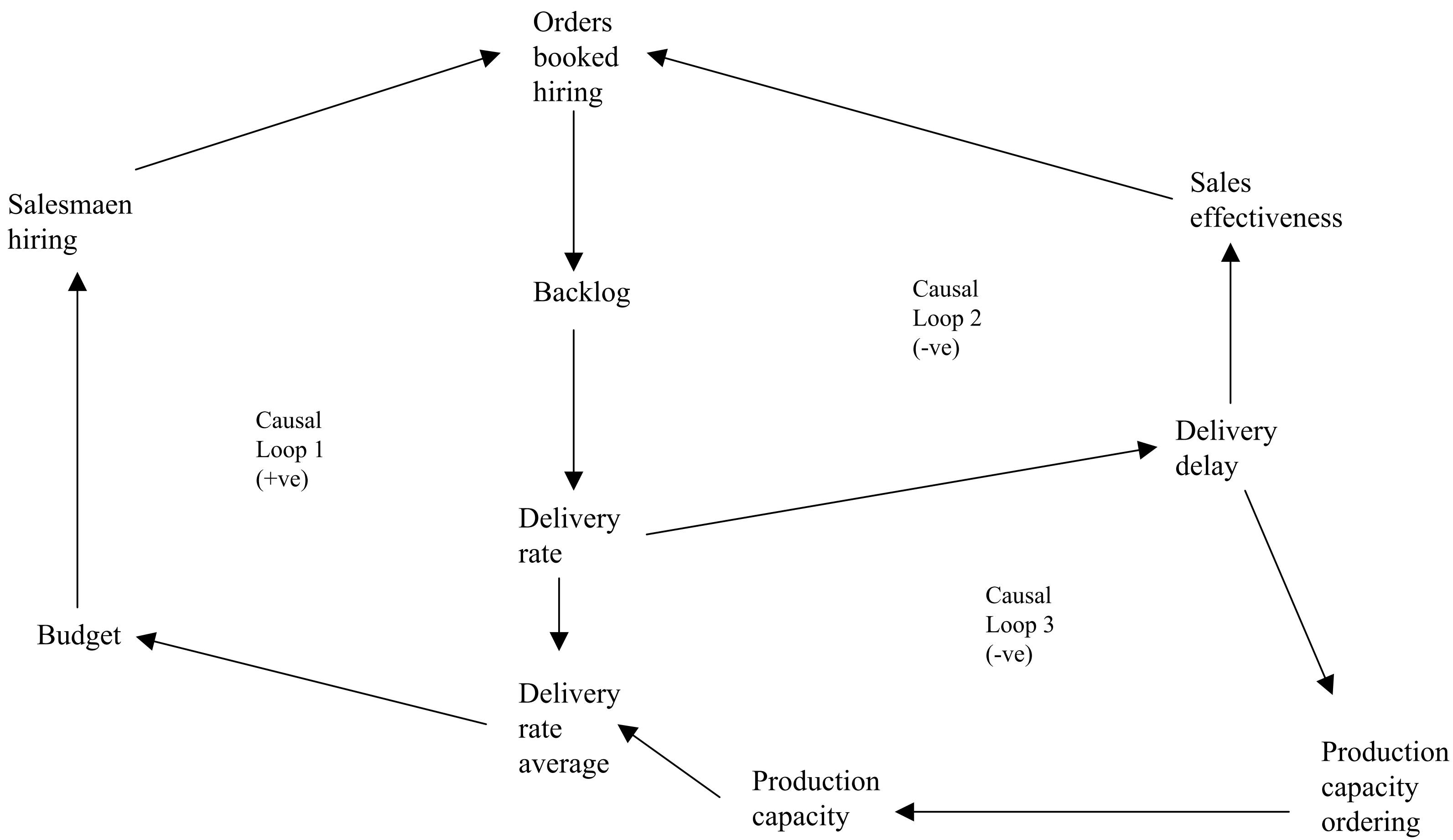


Figure: System Dynamics Modeling of a Business giving Causal Loop Structure for Sales Growth, Delivery Delay and Capacity Expansion

Project Model

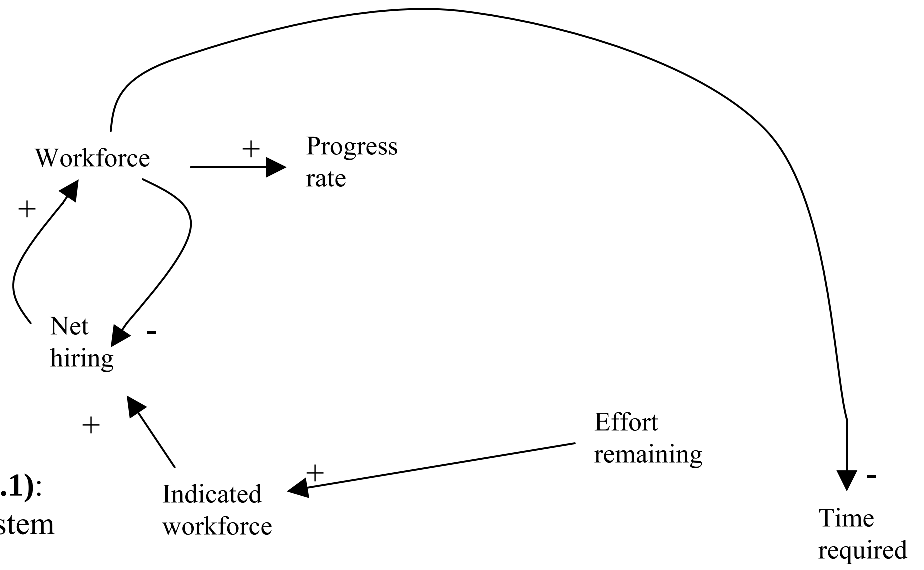


Figure (5.1):
Hiring system

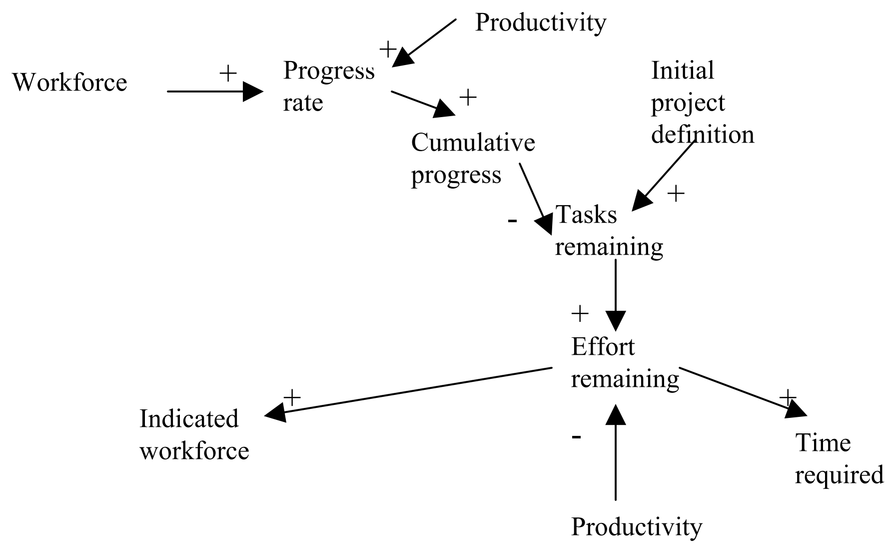


Figure (5.2): A system for "Progress" measurement

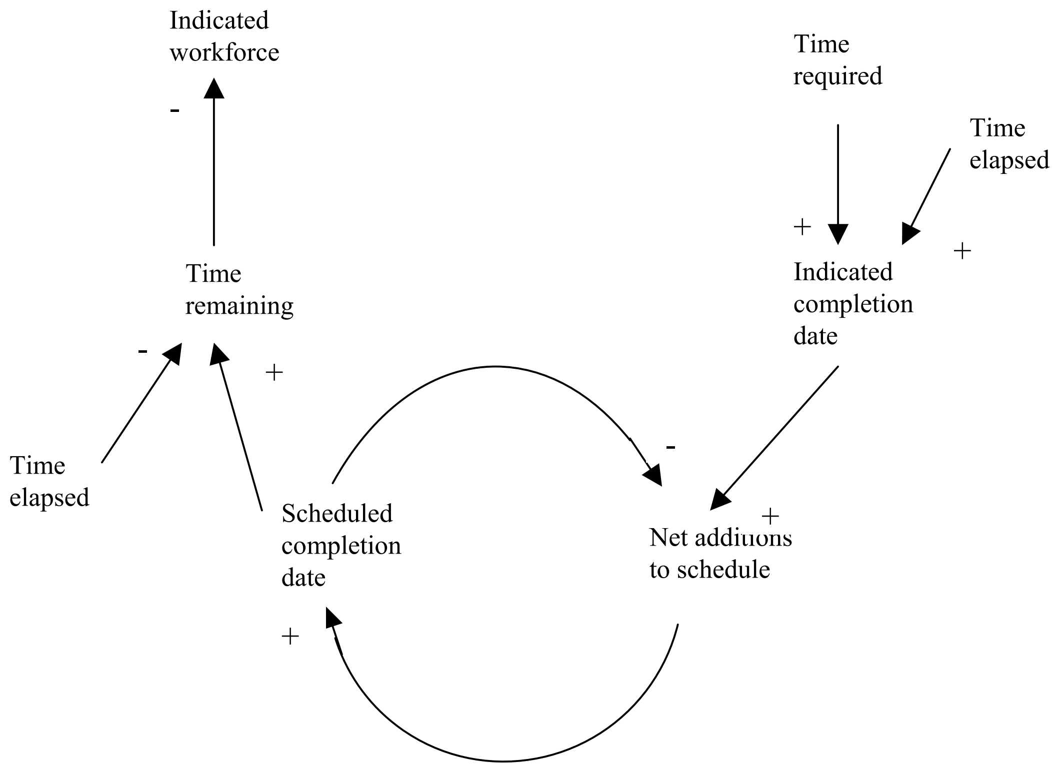
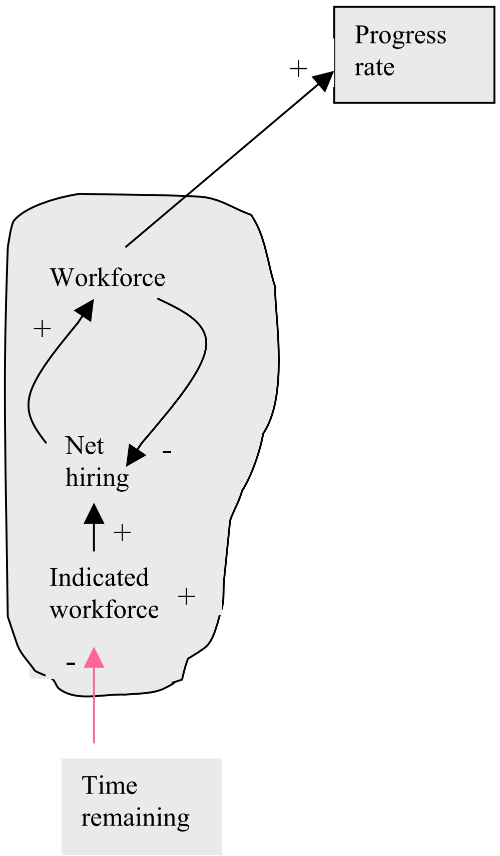
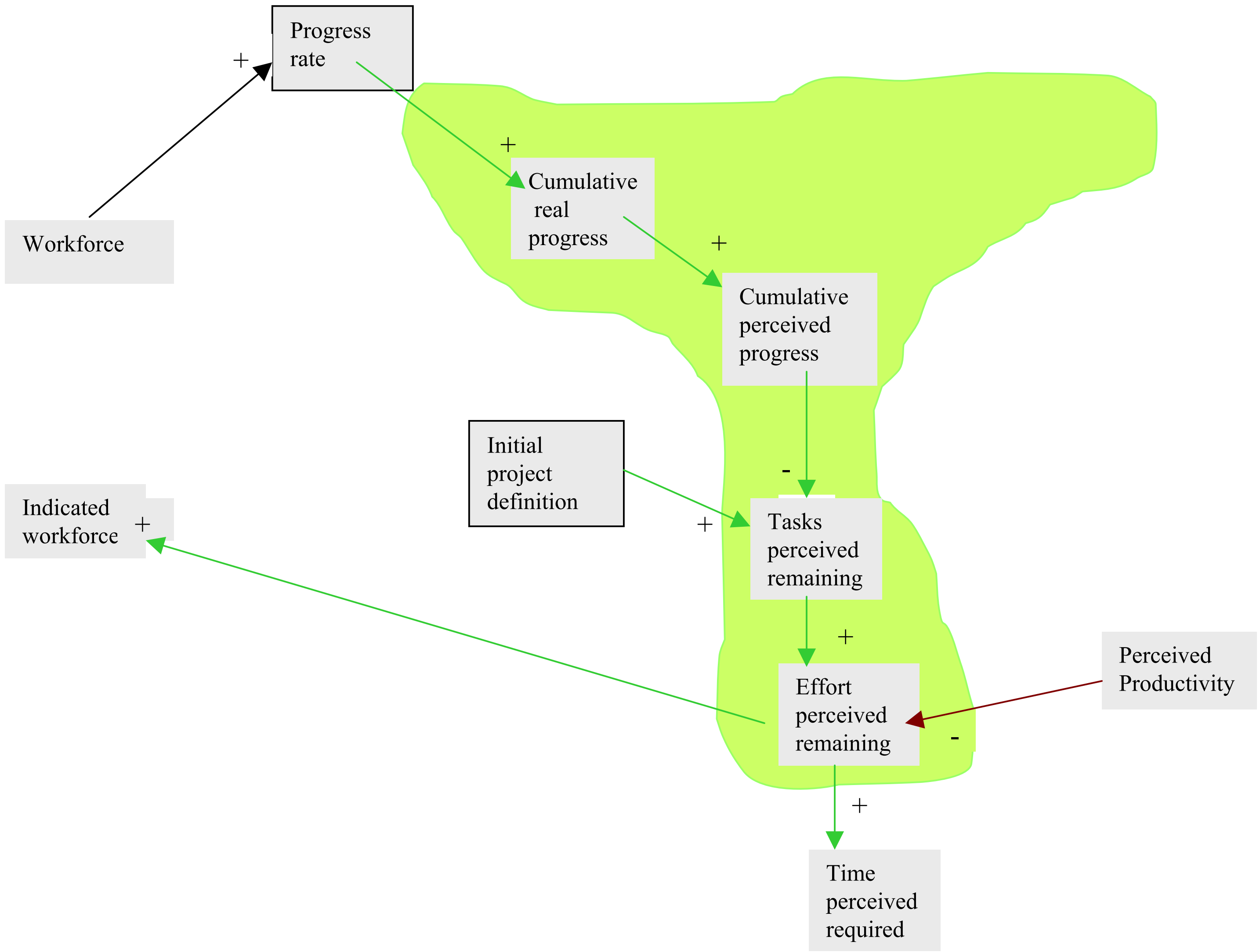
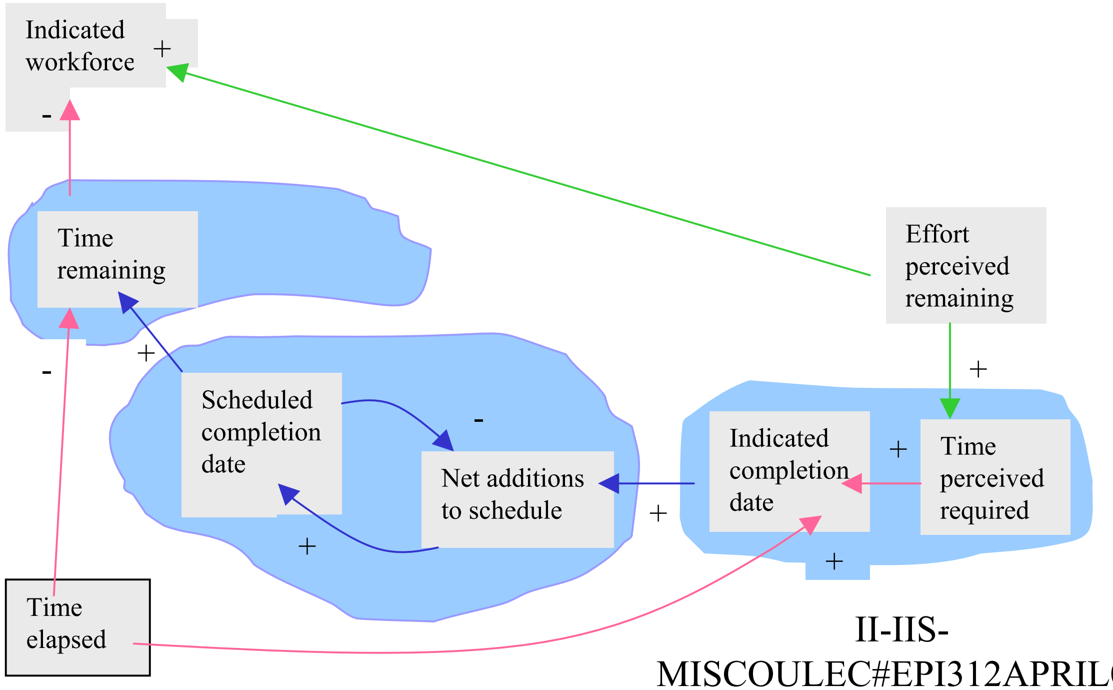


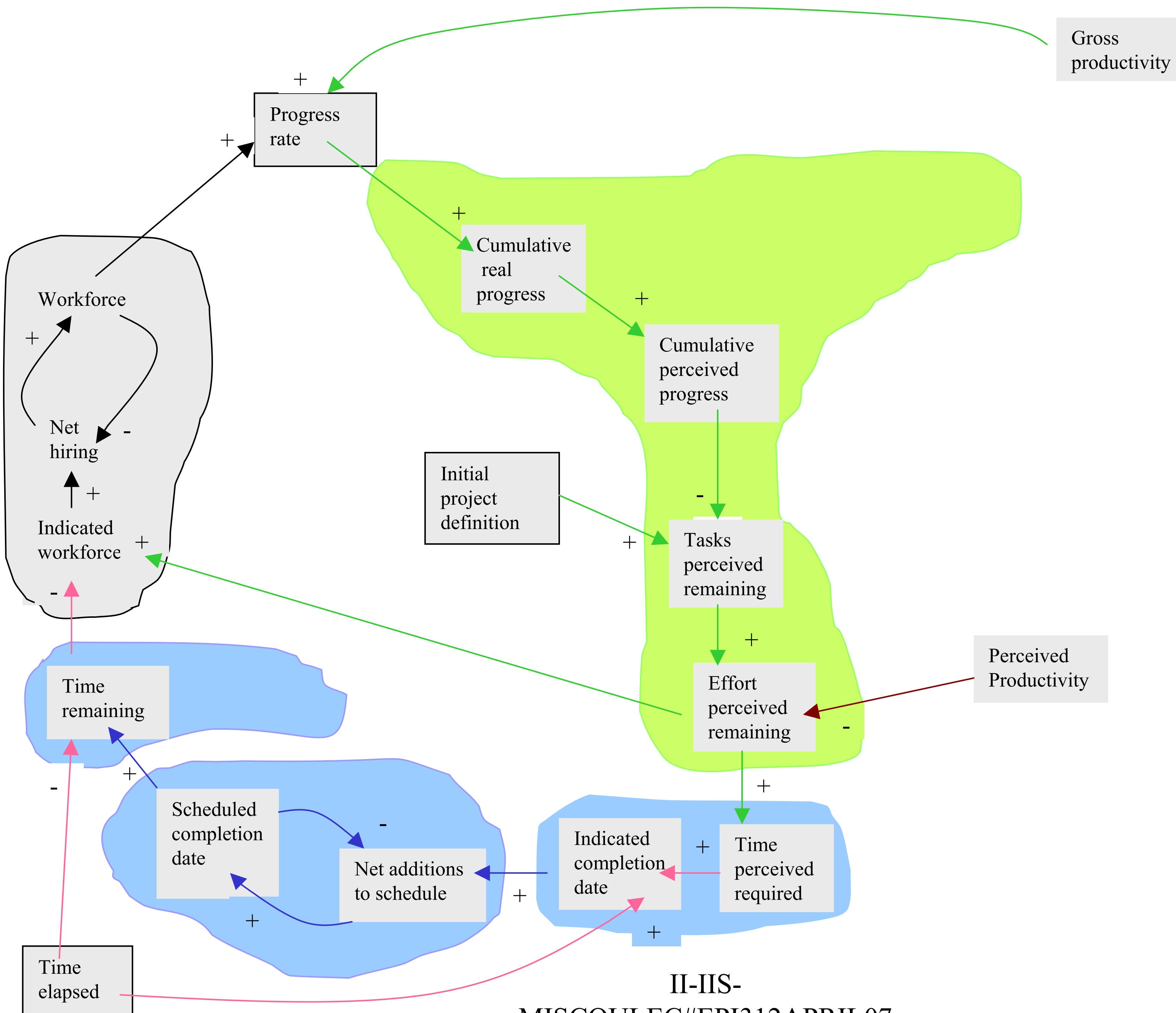
Figure (5.3): A system for assessing “the time required and the time remaining”

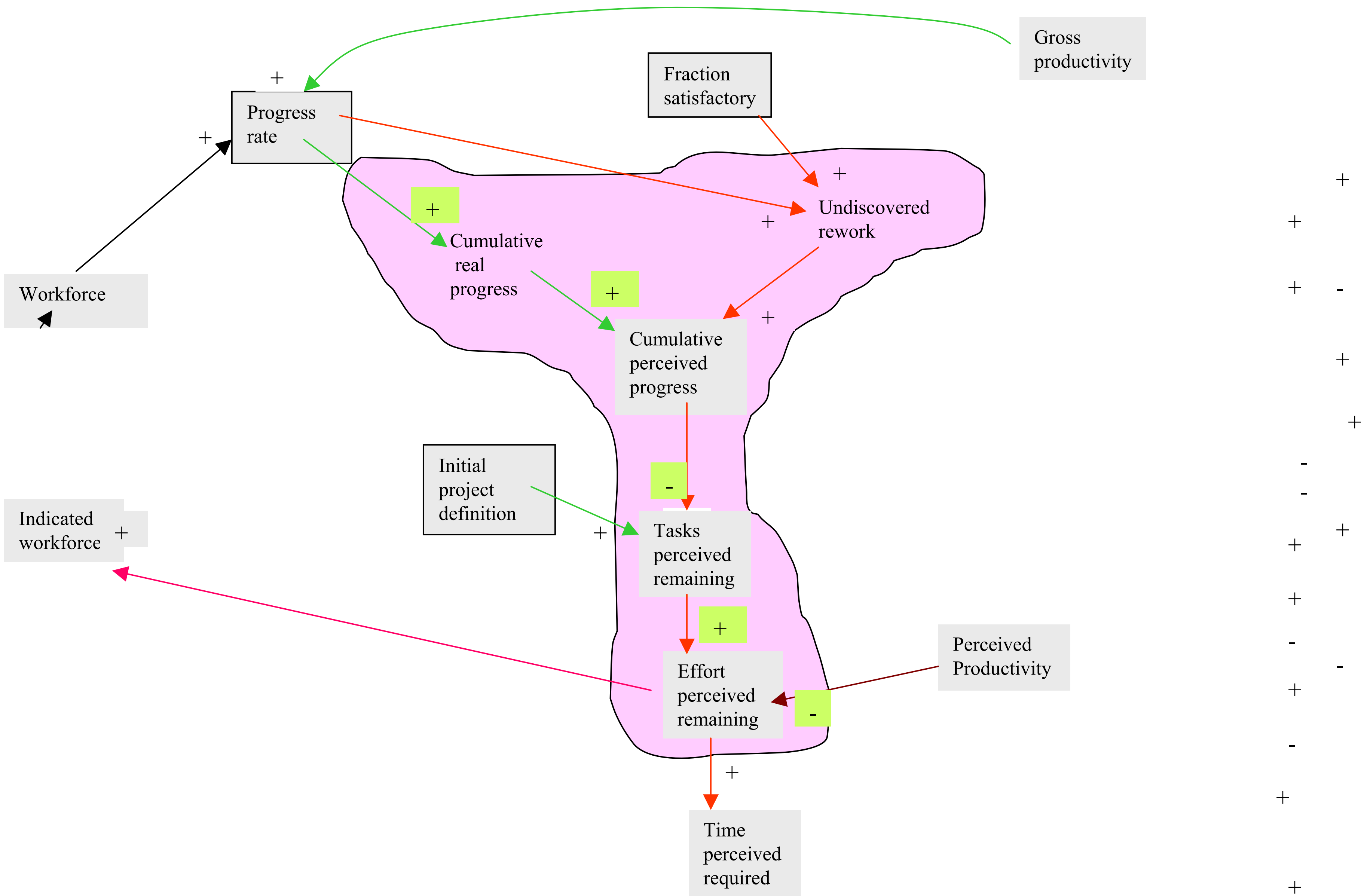

Progress
rate
+
Workforce
+
Net
hiring
-
+
Indicated
workforce
+
-
Time
remaining

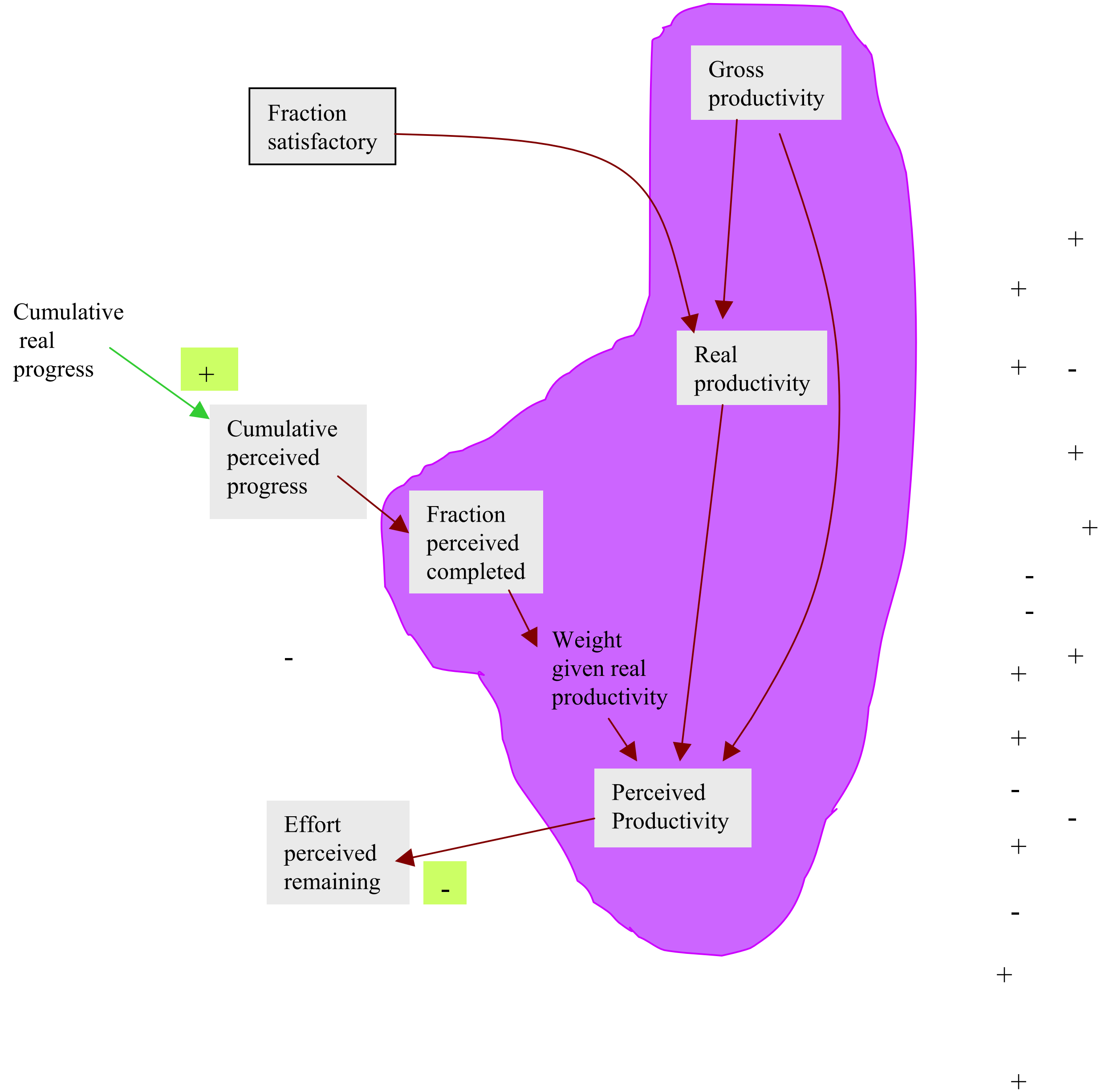

Progress rate
+
Workforce
+
Cumulative real progress
+
Cumulative perceived progress
Initial project definition
-
+
Tasks perceived remaining
Indicated workforce
+
+
Perceived Productivity
Effort perceived remaining
-
+
Time perceived required

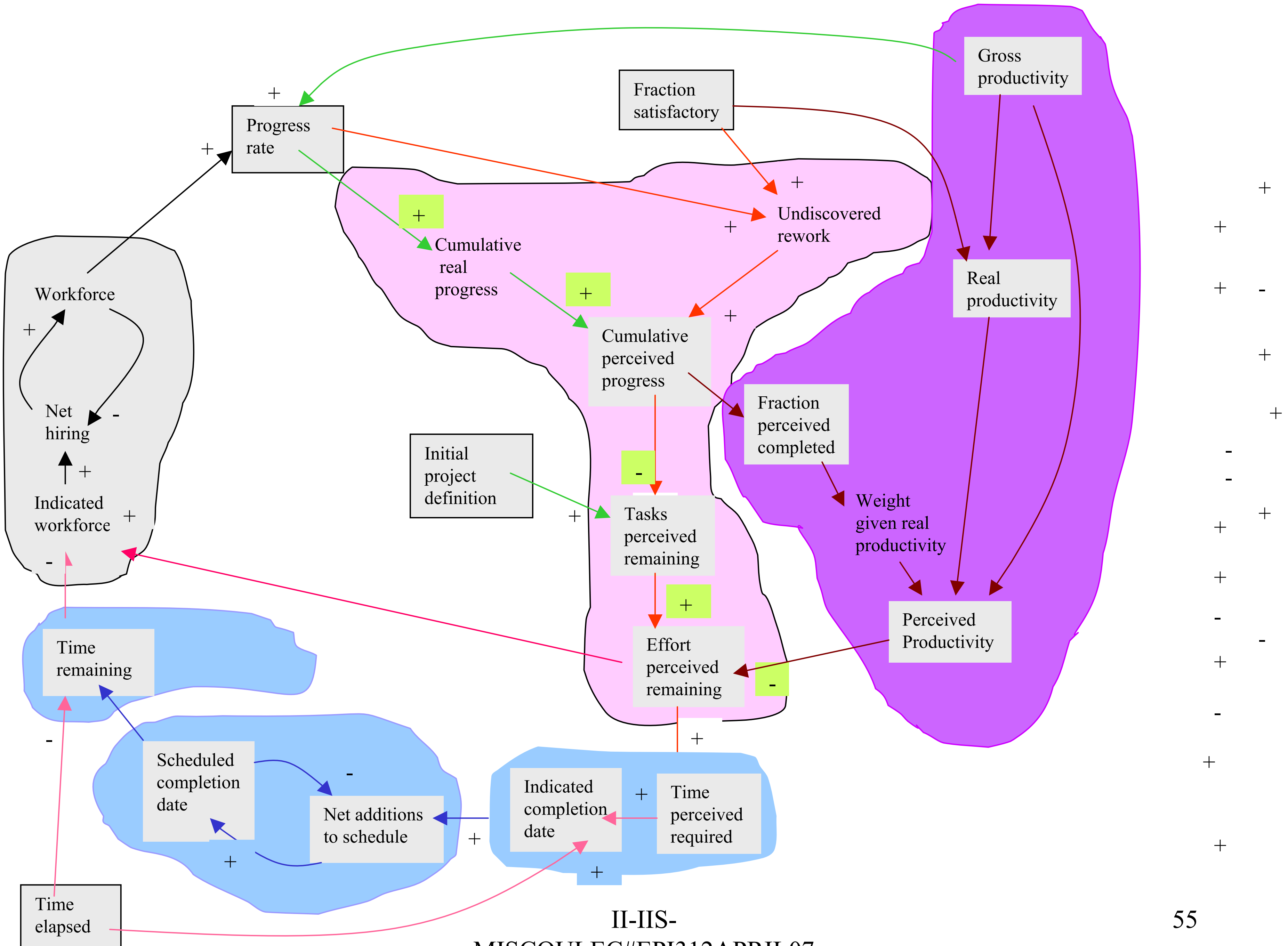


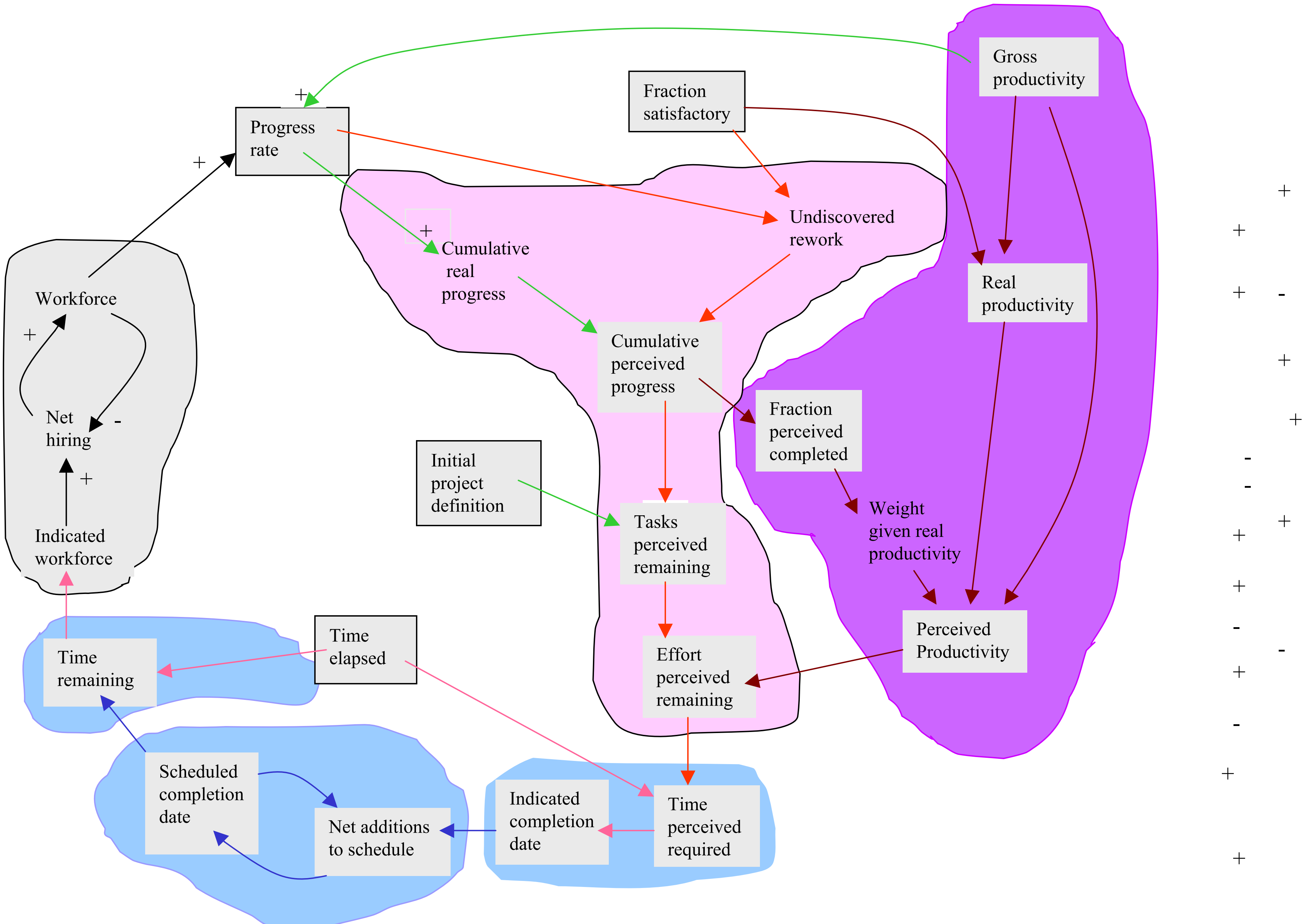
Indicated workforce
+
-
Time remaining
Effort perceived remaining
-
+
+
Scheduled completion date
-
Net additions to schedule
+
Indicated completion date
+
Time perceived required
+
+
Time elapsed


Gross productivity
+
Progress rate
+
+
Cumulative real progress
+
Workforce
+
Cumulative perceived progress
Net hiring
-
+
Initial project definition
-
Indicated workforce
+
+
Tasks perceived remaining
-
+
Perceived Productivity
Effort perceived remaining
-
Time remaining
+
+
-
Scheduled completion date
-
Indicated completion date
+
Time perceived required
Net additions to schedule
+
+
+
Time elapsed


Gross productivity
Fraction satisfactory
Progress rate
Undiscovered rework
Cumulative real progress
Workforce
Cumulative perceived progress
Initial project definition
Indicated workforce
Tasks perceived remaining
Effort perceived remaining
Perceived Productivity
Time perceived required


Fraction satisfactory
Gross productivity
Cumulative real progress
+
Cumulative perceived progress
Real productivity
Fraction perceived completed
Weight given real productivity
Perceived Productivity
Effort perceived remaining
-
-


Gross productivity
Fraction satisfactory
Progress rate
Undiscovered rework
Cumulative real progress
Real productivity
Workforce
Cumulative perceived progress
Fraction perceived completed
Net hiring
Initial project definition
Weight given real productivity
Indicated workforce
Tasks perceived remaining
Perceived Productivity
Effort perceived remaining
Time remaining
Scheduled completion date
Net additions to schedule
Indicated completion date
Time perceived required
Time elapsed


Gross productivity
Fraction satisfactory
Progress rate
Undiscovered rework
Cumulative real progress
Workforce
Real productivity
Cumulative perceived progress
Net hiring
Fraction perceived completed
Initial project definition
Weight given real productivity
Indicated workforce
Tasks perceived remaining
Perceived Productivity
Time elapsed
Effort perceived remaining
Time remaining
Scheduled completion date
Indicated completion date
Time perceived required
Net additions to schedule

Small Sectors of Project Model

- Real progress
- Undiscovered work
- Perceived progress
- Effort perceived remaining
- Hiring
- Scheduling
- The model will be developed around levels and rates identified by considering in turn each of the above small sectors.

- Specifically, the levels and rates will be:
  - Net Hiring Rate and Level Workforce,
  - Real progress rate and Level Cumulative Real Progress
  - Rate of generation of undiscovered rework and Level Undiscovered Work followed by Rate of Detection of Undiscovered Work,
  - Auxiliary variable Perceived Productivity,
  - Rate of Net additions to schedule and level of Scheduled completion date.
  -

Deriving Mathematical Equations for Project Model – An Illustration Example

- We will show equations for Real Progress
- The equations will be as follows:
- A APPRG.K=WF.K * GPROD
- NOTE APPARENT PROGRESS RATE (TASKS/MONTH)
- C GPROD=?
- NOTE GROSS PRODUCTIVITY
- NOTE (TASKS/PERSON/MONTH)
- R RPRG.KL=APPRG.K * FSAT

- NOTE REAL PROGRESS RATE (TASKS/MONTH)
- C FSAT=?
- NOTE FRACTION SATISFACTORY (DIMENSIONLESS)
- Real progress accumulates in the level called cumulative real progress:
- L CPRG.K=CPRG.J + DT* RPRG.JK
- N CRPRG=0

- NOTE   CUMULATIVE REAL PROGRESS (TASKS)
- In this case the level has only an inflow, because real progress, once made, is assumed not to decay. The N equation for CRPRG gives the initial value of cumulative real progress, which ought to be zero at the beginning of a project.
- Please note the forms of the rate, auxiliary, and level equations.

Thank You